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## Bragg spectroscopic interferometer and quantum measurement-induced correlations in atomic Bose–Einstein condensates

M D Lee<sup>1,3</sup>, S Rist<sup>2,3</sup> and J Ruostekoski<sup>1</sup>

<sup>1</sup> School of Mathematics, University of Southampton, Southampton, SO17 1BJ, UK

<sup>2</sup> CNR-SPIN, Corso Perrone 24, I-16152 Genova and NEST, Scuola Normale Superiore, I-56126 Pisa, Italy

E-mail: [mark.lee@soton.ac.uk](mailto:mark.lee@soton.ac.uk)

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**Abstract.** We theoretically analyse the Bragg spectroscopic interferometer of two spatially separated atomic Bose–Einstein condensates that was experimentally realized by Saba *et al* (2005 *Science* **307** 1945) by continuously monitoring the relative phase evolution. Even though atoms in the light-stimulated Bragg scattering interact with intense coherent laser beams, we show that the phase is created by quantum measurement-induced backaction on the homodyne photocurrent of the lasers, opening the possibilities for quantum-enhanced interferometric schemes. We identify two regimes of phase evolution: a running phase regime observed in the experiment of Saba *et al*, which is sensitive to an energy offset and suitable for an interferometer, and a trapped phase regime, which can be insensitive to the applied forces and detrimental to interferometric applications.

<sup>3</sup> These authors contributed equally to this work.

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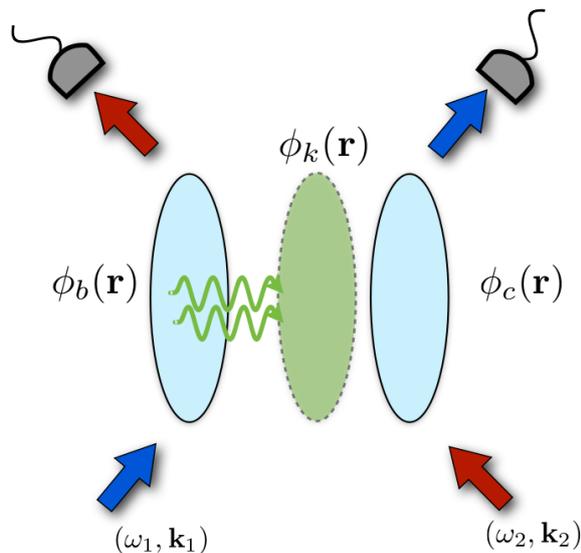
**1. Introduction**

Bragg spectroscopy has become an established spectroscopic tool in ultracold atom experiments [1–6]. In typical setups an intersecting pair of low-intensity pulsed laser beams is used to excite atoms to higher momentum states. The momentum kick experienced by the atoms corresponds to the recoil of a photon upon light-stimulated scattering between the two laser beams. As the spontaneous scattering for off-resonant lasers is negligible and the photons are only exchanged between the directed coherent laser beams, the momentum transfer of the atoms can be measured for specific values of the energy and the momentum. In particular, in a spectroscopic analysis of the many-particle properties of ultracold atoms it is sufficient in the scattering process to describe the light beams classically.

In the experiments by Saba *et al* [7] the relative phase coherence between two Bose–Einstein condensates (BECs) was measured by Bragg scattering atoms between two condensate fragments. Previous Bragg spectroscopy experiments based on time of flight had concentrated on directly detecting the atoms that were transferred to higher momentum states by the laser beams. In the experiment by Saba *et al* [7], however, the strength of the Bragg scattering was measured by monitoring the variations of light intensity in the laser beams by homodyne detection. Due to the correspondence between the light-stimulated scattering of photons between the laser beams and the atoms scattered between two momentum states, the intensity fluctuations are directly proportional to the number of atoms scattered between the condensate fragments.

Saba *et al* [7] measured the light intensity variations of the Bragg beams, which revealed relative phase coherence between the condensates even when the BECs were independently produced and possessed no *a priori* phase information. By theoretically analysing a continuous atom detection process, it has been previously shown that the backaction of quantum measurement of the atomic correlations [8–15], and analogous photon correlations [16, 17], can establish a relative phase between two BECs even when they have ‘never seen each other’ before. It has also been suggested that phase-coherent states of condensates may naturally emerge as robust state descriptions due to dissipative interaction with the environment [18, 19]. With regard to the Bragg spectroscopic interferometer of [7], the question we raise is: how is the phase coherence between the two BECs created, given that the condensates interact with coherent laser beams that can usually be described classically?

Here we analyse a model of the experimental detection scheme [7], illustrated in figure 1, and show that the phase coherence can be built up by continuously monitoring the photocurrent obtained by a homodyne measurement that describes the intensity fluctuations of the laser



**Figure 1.** Our model of the Bragg interferometric measurement of the relative phase between two distant BECs. The two BECs are described by the macroscopic wavefunctions  $\phi_b(\mathbf{r})$  and  $\phi_c(\mathbf{r})$ , and are illuminated by two coherent laser beams. Bragg scattering imparts momentum to atoms from the left condensate, transferring them to the state described by  $\phi_k(\mathbf{r})$ . After an appropriate time the outcoupled atoms will overlap with the right condensate, and the Bragg beams will drive Rabi oscillations between the two atomic clouds. This establishes an optical weak link between the two BECs, and continuous monitoring of the intensity fluctuations in the laser beams measures the phase coherence between the BECs.

beams. We show the rapid establishment of a well-defined relative phase between two independently produced BECs. We identify two distinct regimes of subsequent phase evolution: a running phase and a trapped phase regime. In the running phase regime, the relative phase grows linearly in proportion to the energy offset between the two condensate wells and could be suitable for a weak force detection in interferometric applications [7]. In the trapped phase regime, in the case of a very weak energy offset, the measurement process drives the system close to a dark state where a destructive interference between different scattering paths suppresses the intensity fluctuations of the lasers. In the trapped phase regime, the effect of the energy offset on phase evolution is suppressed, potentially to the detriment of interferometric applications.

Our analysis demonstrates how Bragg spectroscopy can be sensitive to subtle quantum features of ultracold atom systems. Quantum measurement-induced backaction of photocurrent detection on the relative phase coherence of BECs represents a spatially nonlocal entanglement of the laser beams and the relative many-particle state of the atoms. Indeed, the location of the photocurrent detection can be far away from the region of interaction between the coherent laser beams and the atoms. Moreover, one Bragg pulse can be used to entangle the two spatially isolated BECs. A second pulse may then be employed in optical readout of the subsequent evolution dynamics of the measurement-established relative phase coherence between the condensates. An energy offset between the two condensate wells between the subsequent pulses

would result in a detectable phase shift providing potential interferometric applications [7]. Here the phase is determined by a continuous quantum measurement process opening the possibilities for quantum feedback and control methods, e.g. in the generation of sub-shot-noise phase-squeezed states [20, 21]. Such states may be useful in quantum-enhanced metrology for the realization of a high-precision quantum interferometer overcoming the standard quantum limit of classical interferometers [22–25]. Probe field response was also measured recently in the Bragg spectroscopy of condensate excitations in a heterodyne-based detection system, which was able to reach the shot-noise limit [26]. Previous theoretical studies of the effects of continuous monitoring on light scattered from BECs have considered photon counting [16, 17], e.g. in the preparation of macroscopic superposition states [17], and dispersive phase-contrast imaging [27, 28], e.g. in the suppression of heating [28].

This paper is organized as follows. In section 2, we give a short review of the experimental setup of Saba *et al* [7] and the relevant results for this work. We then introduce our basic theoretical model. In section 3, we derive a stochastic differential equation which describes the evolution of the system under the continuous measurement of the scattered light intensity. In section 4, we present our numerical results with a physical interpretation. Finally, some concluding remarks are made in section 5.

## 2. The model and the effective Hamiltonian

An interferometric scheme between two spatially isolated BECs was experimentally realized in [7] without the need for splitting or recombining the two condensate atom clouds. The method was based on the stimulated light scattering of a small fraction of the atoms, only weakly perturbing the condensates and therefore representing an almost nondestructive measurement. Two isolated BECs were prepared in the sites of an unbalanced double-well potential, and illuminated by the same pair of Bragg beams. These beams outcoupled atoms from each well, and the interference between such atoms provided a coupling between the BECs. When outcoupled atoms from one condensate spatially overlapped the second, measurement of the Bragg beam intensity was shown to be sensitive to the relative phase  $\Phi$  between the condensates. In addition, the potential offset between the two wells gave rise to a difference in energies  $\delta\mu$ , which in turn led to a relative phase evolution  $\Phi(t) = \Phi(0) + \delta\mu t/\hbar$ . This was observed as oscillations in the Bragg beam intensity of frequency  $\omega_{\text{osc}} = \delta\mu/\hbar$ , demonstrating that monitoring the Bragg beam intensity directly measured the dynamical evolution of the relative phase between the macroscopic wavefunctions.

In the experiment, a single Bragg pulse established a random relative phase between the two independently produced BECs. If two successive Bragg pulses were applied to the same BEC pair, the relative phase measured by the second pulse was correlated with that detected by the first pulse, indicating that the interaction of the first Bragg beam with the atoms had projected the system into a state with a well-defined relative phase between the condensates.

The key to the method is the weak link established between the BECs by the Bragg laser beams that couple out small atomic samples from the condensates [29]. The coherently driven population dynamics between the BECs is influenced by the relative phase coherence [30, 31], and Bragg scattering may be understood as an interference in momentum space [32]. The specific advantage of the Bragg spectroscopic interference scheme [7] is the nondestructive nature of the detection process, potentially constituting a major advance for interferometric applications since it allows one to probe the evolution of the phase coherence in time by a

continuous measurement process [33–35]. It has also been argued that this setup can be viewed as an analogue of homodyne detection for matter waves [36].

In order to analyse the continuous measurement process of Bragg spectroscopy, we consider the system depicted in figure 1, which is analogous to the experimental setup of Saba *et al* [7]. We assume that the two condensates are initially uncorrelated and that there is no tunnelling between the two spatial regions. As in the experiment, an offset in the trapping potential between the two condensates is accounted for by a difference in chemical potential,  $\delta\mu$ . The condensates are illuminated by two Bragg beams, which impart momentum, kicking atoms out of the traps. The outcoupled atoms propagate from the left condensate to the right and establish an optical weak link between the two macroscopic wavefunctions [29].

For simplicity, in the theoretical analysis we use a single-mode approximation for the condensates and assume that all atoms in the left (right) condensate are in the state  $|b\rangle$  ( $|c\rangle$ ). The atoms in the left (right) condensate are then described by the second quantized field operators  $\hat{\psi}_L(\mathbf{r}) = \phi_b(\mathbf{r})\hat{b}$  ( $\hat{\psi}_R(\mathbf{r}) = \phi_c(\mathbf{r})\hat{c}$ ), which fulfil the usual bosonic commutation relations. Here,  $\hat{b}$  ( $\hat{c}$ ) annihilates an atom in the state  $|b\rangle$  ( $|c\rangle$ ) and  $\phi_{b,c}(\mathbf{r})$  obey the Gross–Pitaevskii equation [37].

Atoms from the BEC in the left well in the state  $|b\rangle$  are transferred by the Bragg beams to the momentum state  $\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$ , where  $\mathbf{k}_j$  are the wavevectors of the Bragg beams. The outcoupled atoms propagate with momentum  $\mathbf{k}$  towards the right BEC in the state  $|c\rangle$ . We take the wavefunction of the outcoupled atoms  $\phi_k(\mathbf{r})$  to be the momentum shifted original wavefunction

$$\phi_k(\mathbf{r}) = \phi_b(\mathbf{r} - \mathbf{r}_L)e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_L)}, \quad (1)$$

where  $\mathbf{r}_L$  ( $\mathbf{r}_R$ ) gives the position of the centre of the left (right) trap. We assume that the momentum kick of the atoms is sufficiently strong, so that the essential characteristics of the continuous quantum measurement process are not obscured by collisions with the remaining trapped atoms, collisions among the outcoupled atoms, and the effect of the trapping potential. We assume that enough time has passed such that the outcoupled atom cloud from the left condensate completely overlaps the right condensate. We therefore neglect the time evolution of the outcoupled cloud while flying from the left to the right trap. In our model, this evolution leads to an additional phase factor which is inconsequential to our findings. We also take the same functional form of the trapping potential for the atoms in the left and right condensates such that  $\phi_b(\mathbf{r} - \mathbf{l}) = \phi_c(\mathbf{r}) \equiv \phi(\mathbf{r})$ , where  $\mathbf{l} = \mathbf{r}_R - \mathbf{r}_L$  is the distance vector between the two potential minima. With these assumptions we find for the effective Hamiltonian

$$H_{\text{eff}} = H_A + H_{\text{AL}} + H_{\text{EM}}, \quad (2)$$

where

$$H_A = \left[ \delta\mu + \frac{\hbar\Omega_1^2}{\Delta} \right] \hat{c}^\dagger \hat{c} + \frac{\hbar\Omega_2^2}{\Delta} \hat{b}_k^\dagger \hat{b}_k + \frac{\hbar\Omega_1\Omega_2}{\Delta} (\hat{c}^\dagger \hat{b}_k + \hat{b}_k^\dagger \hat{c}) \quad (3)$$

describes the Rabi oscillations between the outcoupled atoms and the atoms in the right condensate due to the Bragg beams. Here  $\Omega_j$  are the Rabi frequencies of the Bragg beams,  $\Delta$  is the detuning from the excited state  $|e\rangle$  which couples the two-photon Raman transition between  $|b_k\rangle \leftrightarrow |c\rangle$  and the operator  $\hat{b}_k$  annihilates an outcoupled atom in the momentum shifted state  $|b_k\rangle$  with wavefunction  $\phi_k(\mathbf{r})$ . Hamiltonian (2) is written in the reference frame of the Bragg beams where we assume the two laser frequencies to be equal  $\omega_1 \approx \omega_2 = \omega_L$ . The term

$$H_{\text{EM}} = \hbar \sum_{\lambda} \Delta_{\lambda} \hat{a}_{\lambda}^{\dagger} \hat{a}_{\lambda} \quad (4)$$

takes into account the electromagnetic (EM) vacuum energy, where we have used the standard plane wave decomposition for the EM-field modes. Specifically, the positive frequency component of the vacuum electric field amplitude reads

$$\delta\hat{\mathbf{E}}^+(\mathbf{r}, t) = \sum_{\lambda} \sqrt{\frac{\hbar\omega_{\lambda}}{2\varepsilon_0\mathcal{V}}} \hat{\mathbf{e}}_{\lambda} \hat{a}_{\lambda}(t) e^{i\mathbf{k}_{\lambda}\cdot\mathbf{r}}. \quad (5)$$

Here  $\lambda$  labels a mode of the EM field at wavevector  $\mathbf{k}_{\lambda}$ , polarization  $\hat{\mathbf{e}}_{\lambda} \perp \mathbf{k}_{\lambda}$  and frequency  $\omega_{\lambda} = c|\mathbf{k}_{\lambda}|$ . The velocity of light is denoted by  $c$ , the quantization volume by  $\mathcal{V}$ , the vacuum permittivity is  $\varepsilon_0$  and  $\Delta_{\lambda} = \omega_{\lambda} - \omega_L$ . The operator  $\hat{a}_{\lambda}$  annihilates a photon in mode  $\lambda$ . The total electric field is the sum of the coherent Bragg laser fields  $\mathbf{E}_{\text{in},j}^+(\mathbf{r})$  and the vacuum fields  $\delta\hat{\mathbf{E}}^+(\mathbf{r})$ . The coherent part is responsible for the driving terms in (3), while  $\delta\hat{\mathbf{E}}^+(\mathbf{r})$  provides the coupling of the vacuum modes with the atomic dipoles. We consider off-resonant scattering where the scattering rates for sufficiently large condensates are proportional to the amplitudes of the macroscopically occupied modes due to Bose enhancement, and we neglect scattering to other motional states of the atoms. The coupling between the vacuum modes and the atoms is then given by

$$H_{\text{AL}} = \hbar \sum_{\lambda} (\hat{a}_{\lambda}^{\dagger} \hat{B}_{\lambda} + \hat{B}_{\lambda}^{\dagger} \hat{a}_{\lambda}), \quad (6)$$

where we have introduced the operator

$$\hat{B}_{\lambda} = [(A_1^{\lambda})^* \hat{\sigma}_1 + (A_2^{\lambda})^* \hat{\sigma}_2], \quad (7)$$

with

$$\hat{\sigma}_1 = \hat{c}^{\dagger} \hat{e}, \quad \hat{\sigma}_2 = \hat{b}_k^{\dagger} \hat{e}, \quad (8)$$

and, after adiabatic elimination, the excited state annihilation operator can be written as

$$\hat{e} = \left( \frac{\Omega_1}{\Delta} \hat{c} + \frac{\Omega_2}{\Delta} \hat{b}_k \right). \quad (9)$$

We have defined

$$A_j^{\lambda} = \sqrt{\frac{\hbar\omega_{\lambda}}{2\varepsilon_0\mathcal{V}}} (\mathbf{d}_j^- \cdot \hat{\mathbf{e}}_{\lambda}) \int d\mathbf{r} |\phi(\mathbf{r})|^2 e^{-i(\mathbf{k}_j - \mathbf{k}_{\lambda})\cdot\mathbf{r}}, \quad (10)$$

where the factor outside the integral is the coupling strength between the atomic dipoles and the EM-field mode  $\lambda$  [38]. Here the matrix elements of the dipole moment operator  $\hat{d}$  for the transition are denoted by  $\mathbf{d}_1^- = \langle c|\hat{d}|e\rangle$ ,  $\mathbf{d}_2^- = \langle b_k|\hat{d}|e\rangle$ .

### 3. Continuous homodyne measurement

We consider the condensate and the outcoupled atomic cloud together with the driving fields as an open quantum system and eliminate the vacuum EM-field modes. The aim of our treatment is to compute the evolution of the reduced system under continuous measurement of the light intensity of the Bragg beams. The intensity of the beam  $j$  is given by

$$I_j = 2c\varepsilon_0 \langle \hat{\mathbf{E}}_j^-(\mathbf{r}, t) \hat{\mathbf{E}}_j^+(\mathbf{r}, t) \rangle. \quad (11)$$

Here the total electric field amplitude of each Bragg beam is given by the sum of the coherent driving laser field and the field  $\delta\hat{\mathbf{E}}_j^+(\mathbf{r}, t)$  due to scattering in the direction of the beam  $j$

$$\hat{\mathbf{E}}_j^+(\mathbf{r}, t) = \mathbf{E}_{\text{in},j}^+(\mathbf{r}) + \delta\hat{\mathbf{E}}_j^+(\mathbf{r}, t). \quad (12)$$

Assuming that the amplitude of the scattered field is small compared to the applied laser field, the measured intensity is approximately

$$I_j \simeq 2c\epsilon_0(\langle \mathbf{E}_{\text{in},j}^- \mathbf{E}_{\text{in},j}^+ \rangle + \langle \mathbf{E}_{\text{in},j}^- \delta\hat{\mathbf{E}}_j^+ \rangle + \langle \delta\hat{\mathbf{E}}_j^- \mathbf{E}_{\text{in},j}^+ \rangle), \quad (13)$$

where the last two terms give rise to fluctuations in the intensity incident on the detector  $j$ .

We may now solve the intensity fluctuations by calculating the scattered field amplitude from the effective system Hamiltonian. From the Heisenberg equation of motion for  $\hat{a}_\lambda$ , one finds that

$$\hat{a}_\lambda(t) = \hat{a}_\lambda(0)e^{-i\Delta_\lambda t} - i \int_0^t dt' e^{-i\Delta_\lambda(t-t')} ((A_1^\lambda)^* \hat{\sigma}_1(t') + (A_2^\lambda)^* \hat{\sigma}_2(t')), \quad (14)$$

with  $\langle \hat{a}_\lambda(0) \rangle = 0$ . Inserting (14) into (5) and defining  $\mathbf{q}_j = k_L \mathbf{n} - \mathbf{k}_j$  as the change of the wavevector of light upon scattering with  $k_L = \omega_L/c$  [39], we then obtain two contributions from the vacuum field, one for each beam

$$\delta\hat{\mathbf{E}}_j^+(\mathbf{r}, t) \simeq \frac{k_L^2 e^{ik_L r_D}}{4\pi \epsilon_0 r_D} \mathbf{n} \times (\mathbf{n} \times \mathbf{d}_j^-) \hat{\sigma}_j(t) \int d\mathbf{r}' |\phi(\mathbf{r}')|^2 e^{i\mathbf{q}_j \cdot \mathbf{r}'}. \quad (15)$$

The spatial integral over the wavefunction  $\phi(\mathbf{r}')$  enforces an approximate momentum conservation, so that the photons are dominantly scattered into a cone centred at  $\mathbf{q}_j = 0$  in the direction of the laser beam  $j$ . In deriving (15) we made the expansion  $|\mathbf{r} - \mathbf{r}'| = r_D - \mathbf{n} \cdot \mathbf{r}'$ , with  $\mathbf{n}$  being the unit vector that points from the scattering region to the detector,  $r_D$  is the distance between the detector and a representative point at the origin of the scattering region. Due to the normalization of the wavefunction we finally find for the scattered electric field in the two outgoing beams

$$\delta\hat{\mathbf{E}}_j^+(\mathbf{r}, t) = \frac{k_L^2 e^{ik_L r_D}}{4\pi \epsilon_0 r_D} \mathbf{n} \times (\mathbf{n} \times \mathbf{d}_j^-) \hat{\sigma}_j(t). \quad (16)$$

The atomic operator associated with the spontaneous emission of a photon into beam  $j$  in (16) is given by  $\hat{\sigma}_j$ . The master equation which describes the evolution of the reduced density matrix after elimination of the vacuum field modes then reads [40]

$$\dot{\hat{\rho}}(t) = \frac{i}{\hbar} [\hat{\rho}, \hat{H}_A] - \sum_{j=1,2} \frac{\gamma_j}{2} (\hat{\sigma}_j^\dagger \hat{\sigma}_j \hat{\rho} + \hat{\rho} \hat{\sigma}_j^\dagger \hat{\sigma}_j - 2\hat{\sigma}_j \hat{\rho} \hat{\sigma}_j^\dagger). \quad (17)$$

Here  $\gamma_j$  is the rate of spontaneously scattered photons, and is related to the total spontaneously scattered light intensity  $\delta I = 2c\epsilon_0 \langle \delta\hat{\mathbf{E}}_j^-(\mathbf{r}, t) \delta\hat{\mathbf{E}}_j^+(\mathbf{r}, t) \rangle$  via [17]

$$\frac{1}{\hbar k_L c} \int d\Omega r_D^2 \delta I_j = \gamma_j \langle \hat{\sigma}_j^\dagger \hat{\sigma}_j \rangle, \quad (18)$$

where the angular integral is over the scattering cone of beam  $j$ . The operators associated with the light field amplitude of the beam  $j$  read

$$\hat{C}_j = \sqrt{\gamma_j} (\alpha_j + \hat{\sigma}_j), \quad (19)$$

where  $\alpha_j$  is proportional to the amplitude of the coherent laser beam with wavevector  $\mathbf{k}_j$ . The intensity is then proportional to  $\langle \hat{C}_j^\dagger \hat{C}_j \rangle$ . The leading contribution comes from the coherent intensity  $\propto \gamma_j |\alpha_j|^2$  (corresponding to the first term in (13)) and the intensity fluctuations are dominated by the terms  $\gamma_j (\alpha_j \langle \hat{\sigma}_j \rangle + \text{c.c.})$  (corresponding to the second and the third term in (13)). Extending the treatment of Wiseman and Milburn [41] to our setup, one finds that the evolution of the system under the continuous monitoring of light intensity can be described by the stochastic differential equation

$$|\psi(t+dt)\rangle = \left( 1 - \frac{i}{\hbar} \hat{H}_A dt + \sum_j \left[ \frac{\gamma_j}{2} \hat{\sigma}_j^\dagger \hat{\sigma}_j dt + 2\gamma_j X_j dt + \hat{\sigma}_j \sqrt{\gamma_j} dW_j \right] \right) |\psi(t)\rangle, \quad (20)$$

where

$$X_j = \frac{1}{2} (\hat{\sigma}_j + \hat{\sigma}_j^\dagger). \quad (21)$$

Here,  $dW_j$  is a Wiener increment with zero mean  $\langle dW_j \rangle = 0$  and  $\langle (dW_j)^2 \rangle = dt$ , which appears as a result of the continuous measurement process. Keeping terms to lowest order in the fluctuations, one finds an expression for the photocurrent in essence equivalent to (13)

$$i_j^{\text{phot}}(t) = \gamma_j \alpha_j^2 + \alpha_j (2\gamma_j \langle X_j \rangle + \sqrt{\gamma_j} \xi_j(t)). \quad (22)$$

Here,

$$\xi_j(t) = \frac{dW_j}{dt} \quad (23)$$

represents Gaussian white noise [41] and arises from the open nature of our quantum system.

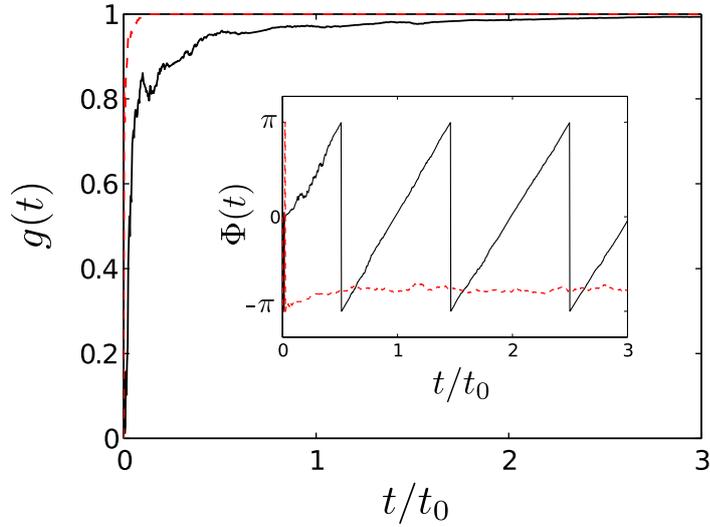
#### 4. Numerical results

In order to study the effect of the homodyne photocurrent measurements on the system, we numerically integrate (20) using the Milstein algorithm [42]. As the initial state in the numerical simulations we take a pure number state in each condensate, with no well-defined phase between them, and the incident Bragg laser beams are taken to be classical coherent states. The relative phase between the condensates as a function of time may then be calculated as  $\Phi(t) = \arg(\langle \hat{c}^\dagger \hat{b} \rangle)$ . We define a measure for the strength of the phase coherence between the condensates by the absolute value of the normalized phase coherence

$$g(t) = \frac{|\langle \hat{c}^\dagger \hat{b} \rangle|}{\sqrt{\langle \hat{c}^\dagger \hat{c} \rangle \langle \hat{b}^\dagger \hat{b} \rangle}}. \quad (24)$$

A value of  $g(t)$  close to 1 indicates a high degree of relative phase coherence, while condensates with no relative phase information have  $g(t) \simeq 0$ .

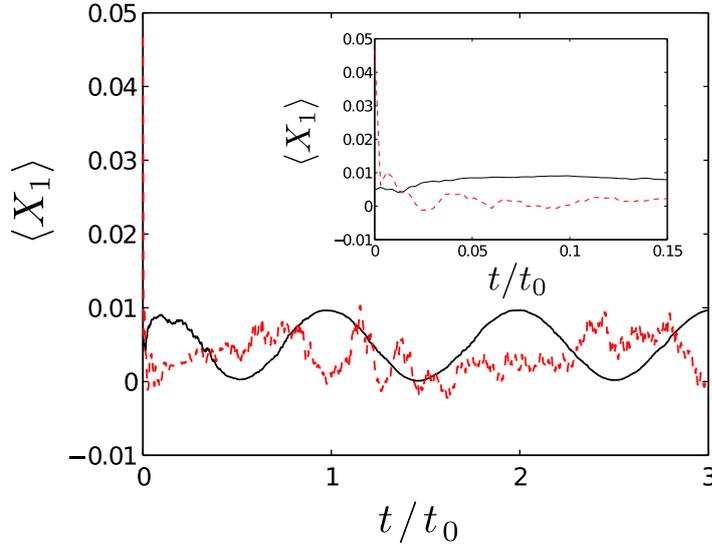
In figure 2, we plot the time evolution of the coherence and the relative phase for two different values of the detuning  $\Delta$ . No well-defined relative phase exists at early times, the coherence starts at zero and  $\Phi(t)$  shows large random fluctuations with time. As the continuous measurement proceeds the coherence builds rapidly, leading to a well-defined relative phase with a stable value. Once established, we then see two different regimes of behaviour at longer times. For large values of the detuning  $\Delta = 100\gamma_1$  we see a running phase behaviour: once well established with a value which is random for each individual run, the phase grows



**Figure 2.** Coherence  $g(t)$  as a function of time  $t$  in units of  $t_0 = 2\pi\hbar/\delta\mu$ . The simulation is done for a total of  $N = 100$  atoms which are initially distributed equally between the states  $|b_k\rangle$  and  $|c\rangle$ . Parameters were chosen to be  $\gamma_1 = \gamma_2 = 10^5/t_0$  and  $\sqrt{\Omega_1\Omega_2} = 10^3/t_0$ . The black solid line corresponds to  $\Delta = 100\gamma_1$  and the red dashed line to  $\Delta = 10\gamma_1$ . In the inset we show the evolution of the phase  $\Phi(t)$  as a function of time  $t$  in units of  $t_0$ .

linearly in time with a rate proportional to the difference in energies between the condensates  $\Phi(t) \sim \delta\mu t/\hbar$ . From (22) we note that the measured photocurrent from the two Bragg beams is essentially proportional to the quadrature  $\langle X_j \rangle$  after subtracting the background current, and the corresponding time evolution of  $\langle X_j \rangle$  is shown in figure 3. In the running phase regime, the quadrature exhibits well-defined oscillations with frequency  $\omega_{\text{osc}} = \delta\mu/\hbar$ , and this corresponds to the experimental measurements obtained by Saba *et al* [7]. Such oscillations thus give an interferometric measurement of the relative phase evolution, sensitive to any accumulated phase shift due to an energy offset between the distant condensates. An interferometer of this type could be used, for example, to detect a weak force applied to one of the condensates.

Choosing a smaller value for the detuning  $\Delta = 10\gamma_1$ , we find a very different long-time behaviour. Once again the phase fluctuates as coherence is established, although this occurs on a much faster timescale. This can be understood from the fact that the phase is established as a result of the intensity fluctuations in the laser beams, which are enhanced by decreasing the detuning  $\Delta$ . Unlike the running phase case, once firmly established the phase now locks to an almost constant value near  $\pi$ . This trapped phase state has the two condensates almost entirely out of phase, leading to destructive interference in the oscillations between the states  $|b_k\rangle$  and  $|c\rangle$  and resulting in a state analogous to a dark state. The corresponding quadrature  $\langle X_1 \rangle$  therefore exhibits merely random fluctuations which would not be suited to an interferometric type experiment. Note that although one would expect the amplitude of these random fluctuations to be suppressed compared to the coherent oscillations of the running phase regime, and this is indeed the case, the two different values of detunings used here do not allow such a direct comparison in figure 3.



**Figure 3.** Mean value of the quadrature  $\langle X_1 \rangle$  as a function of time  $t$ . The black solid line corresponds to  $\Delta = 100\gamma_1$  and the red dashed line to  $\Delta = 10\gamma_1$ . The other parameters are the same as in figure 3. The inset shows a magnified view of the initial behaviour.

The two different regimes of behaviour resemble the ac-Josephson and self-trapping behaviours seen in double-well condensates [43–45], although we emphasize that here coupling occurs due to the nonlocal measurement process induced by the Bragg beams. The trapped phase behaviour is more akin to a dark state however, due to the lack of any nonlinearity in the Hamiltonian which is required for macroscopic self-trapping. The different regimes may be understood if we assume that a well-defined phase and population can be associated with each condensate, and neglect any processes other than those included in  $H_A$ , leaving a two-mode model similar to that considered in [43]. The trapped phase regime then occurs with a stable relative phase difference of  $\pi$  when

$$\delta\mu = 2\hbar \frac{\Omega_1\Omega_2}{\Delta} \frac{|z|}{\sqrt{1-z^2}}, \quad (25)$$

where  $z = (N_k - N_c)/(N_k + N_c)$  is the relative population difference, where  $N_{k(c)}$  is the population in state  $|b_k\rangle(|c\rangle)$ . The trapped phase regime therefore requires either  $\delta\mu/\hbar \sim \Omega_1\Omega_2/\Delta$  or a large population imbalance. This is in agreement with our results, where the larger value of detuning has  $\delta\mu/\hbar \gg \Omega_1\Omega_2/\Delta$ , and the initial population balance is not extreme. The trapped phase condition is then not satisfied and we observe a running phase behaviour akin to the ac-Josephson effect. Note, however, that dissipation is vital for establishing the relative phase in the first place, and has the effect of shifting the relative phase in the trapped regime away from  $\pi$  in figure 2. The model (25) does not specify the role of spontaneous scattering which determines the rate at which the system is driven towards the trapped phase state.

In the experiment by Saba *et al* [7], the system typically contained of the order of  $10^6$  atoms, although the numbers outcoupled would be only a small fraction of this. Numerical simulations in our basis for such large numbers are prohibitively slow, so here we have typically used a total atom number of 100. Our results show no significant dependence on atom number however,

and we expect our results to give a good qualitative comparison with the physics exhibited in the experiment. We have so far discussed our results in dimensionless units; in order to give a specific example we may take  $\gamma_{1,2}$  to be of the order of  $2\pi \times 10$  MHz, for instance. Our results then correspond to the values  $\delta\mu/\hbar \sim 2\pi \times 630$  Hz and  $\sqrt{\Omega_1\Omega_2} \sim 2\pi \times 0.45$  MHz.

Parameters used in the experiment by Saba *et al* [7] were  $\delta\mu/\hbar \sim 2\pi \times 1$  kHz,  $\Delta = 2\pi \times 1$  GHz,  $\sqrt{\Omega_1\Omega_2} \sim 2\pi \times 0.45$  MHz. This yields a ratio  $\eta \equiv 2\hbar\Omega_1\Omega_2/(\Delta\delta\mu) \sim 0.4$ . In order for the trapped phase regime to be observed the population imbalance would then be required to satisfy  $z \approx 0.92$ . In the experiment, the actual population in the momentum-shifted state ( $|b_k\rangle$ ) that overlapped the second condensate ( $|c\rangle$ ) was of the order of  $2 \times 10^4$  atoms during the coupling. The corresponding population imbalance was in excess of 0.96, and hence did not satisfy condition (25) for the trapped phase behaviour. The observed running phase behaviour in the experiment is therefore consistent with our model.

## 5. Concluding remarks

Bragg spectroscopy was used in [7] to measure the relative phase between two initially uncorrelated BECs. By studying a simplified model containing the essential ingredients of the experiment, we have demonstrated how the homodyne measurement process builds up a coherent relative phase between the two condensates. This quantum measurement-induced backaction entangles the two macroscopic many-body states even though the measurement location can be far away from the region of interactions.

Following the establishment of a coherent phase, we have identified two distinct behaviours under continual subsequent measurement. With a larger atom–laser detuning  $\Delta$  or a large initial energy imbalance  $\delta\mu$ , we reproduce the experimental findings of [7], with the measured photon flux exhibiting oscillations at a frequency corresponding to the energy offset of the separated condensates. In this case, once the coherence and a random-valued phase are established it evolves linearly in time  $\Phi(t) \sim \delta\mu t/\hbar$ . Measurable oscillations in the laser beam intensity mean that this state has applications in quantum-enhanced interferometry, and the measurement backaction could potentially be used further to implement feedback mechanisms [21]. By choosing a smaller atom–laser detuning, we found instead that the system stabilized to a trapped phase state with the condensate relative phase fixed at almost  $\pi$ , while the scattered light intensity showed only random fluctuations. A semiclassical model can qualitatively describe the difference between these two regimes, with the trapped phase behaviour occurring when (25) was satisfied.

When atoms from two initially uncorrelated condensates overlap, we have shown that Bragg coupling and continuous homodyne measurement can rapidly establish a well-defined relative phase. A closely related experiment [46] has been performed using ultra-slow light pulses, in which optical information was coherently transported between two spatially separated condensates by a travelling matter wave. An ultra-slow light pulse was stopped in the first condensate, creating a dark-state superposition between two atomic internal states. Upon stopping the pulse, one of the internal states received a momentum kick, and outcoupled atoms in this state passed through a second distant condensate. By illuminating the second condensate with a coupling laser it was possible to revive the initial light pulse even when the BECs were independently produced. In the case when the condensates had been prepared separately, the rapid establishment of a coherent phase in a manner similar to that described in this paper explains the recovery of the light pulse.

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