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Optical super-oscillations: sub-wavelength light focusing and super-resolution imaging

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Received 17 July 2013, accepted for publication 9 August 2013
Published 10 September 2013
Online at stacks.iop.org/JOpt/15/094008

Abstract
Optical super-oscillations, first predicted in 1952 and observed in 2007, offer a promising route to optical super-resolution imaging and show potential for manufacturing with light and data-storage applications such as direct optical recording and heat assisted magnetic recording. We review the history and basic physics behind the phenomenon of super-oscillation and its application in optics. We overview recent results in creating optical super-oscillations using binary masks, spatial light modulators and planar metamaterial masks. We also investigate the limits and competitiveness of super-oscillatory imaging.

Keywords: super-resolution, microscopy, super-oscillations

1. Introduction

Looking at objects is one of the most intuitive ways of getting information about the world around us. Since the days of Antonie van Leeuwenhoek’s discovery of single celled organisms using his home built microscopes, advances in imaging technology have driven scientific and technological advances across all fields. In the four centuries since, the Nobel Prize has been awarded four times for the development of imaging techniques. For many applications, the simplest and most popular imaging technique is optical microscopy. Unfortunately standard optical microscopy is limited in the resolution it can achieve to approximately the wavelength of the light used. Despite this, optical microscopes remain the tool of choice for visualizing cells in the biological sciences and for inspection of microscale components. Numerous techniques exist to beat the diffraction limit and produce super-resolution optical microscopes (see, for example, the reviews [1–4] and references therein). In this review we discuss a recently developed technique for super-resolution focusing and imaging using optical super-oscillations.

Ernst Abbe, in 1873, discovered that the resolution of an optical microscope is fundamentally limited, not by the quality of the instrument, but by the diffraction of light. For an ideally constructed microscope, the resolution, \(d\), is limited by the wavelength of light, \(\lambda\), and the numerical aperture (NA) of the objective lens to \(d = \frac{1.22 \lambda}{2 \text{NA}}\). For a microscope operating in air, the numerical aperture is limited to 1, and the resolution cannot be better than about 0.6\(\lambda\), although some improvements in resolution can be achieved by imaging in immersion media of higher refractive index [5, 6].

To see the origin of this diffraction limit, we can consider a somewhat simplified picture of light scattering by an object. When illuminated by light, structures with a size greater than \(d\) scatter light predominantly into waves that propagate away from the structures and can be captured by an optical microscope. Structures in the object with size less than \(d\) scatter light predominantly into evanescent waves that are localized in the vicinity of the object and decay exponentially away from it with a characteristic length on the scale of the wavelength. These evanescent waves, that contain fine information about the object, therefore, cannot be collected by a conventional microscope and do not contribute to...
the recorded image. An obvious way, therefore, to form a super-resolved image is to capture the evanescent waves from the near-field of the sample.

The simplest way to make use of evanescent waves in imaging is to simply place a recording medium in direct contact with the object to be imaged—known as contact printing or contact lithography. The process was allegedly invented in the 1820s by Joseph-Nicéphore Niépce to reproduce photographic images, though without the intention of achieving sub-wavelength resolution. An example of an image produced in this way is shown in figure 1. Contact lithography is still in use in micro- and nanofabrication to reproduce photomasks in photoresist with resolutions up to $\frac{\lambda}{20}$ [7].

The capture of evanescent waves was proposed by Synge as early as 1928 [8] but was not realized as an imaging technique until scanning near-field optical microscopy (SNOM) was developed in 1984 [9, 10]. In figure 2(a) we show a schematic of collection mode aperture SNOM, showing a scanning tip passing over a surface, in the near-field, and collecting the light transmitted through a sub-wavelength hole. In this way, the resolution is determined not by the wavelength of light, but by the tip aperture size and distance from the object. To achieve high resolution, a small tip that is scanned within the near-field of the surface is needed—in the optical regime the tip aperture and tip to surface distance must be of the order of 10 nm.

Another approach to super-resolution using evanescent waves is the negative index superlens. Victor Veselago [11] first showed that a simple slab of material with negative refractive index would function as a lens in 1968 although it was John Pendry who first proposed that such a superlens would reconstruct evanescent waves and lead to perfect imaging [12]. The principle of the lens is illustrated in figure 2(b): the slab of negative index material acts to ‘cancel out’ the effects of propagation though the physical space, thus imaging evanescent fields from the object to the image. This allows perfect imaging. Although the principle of negative index superlenses is already experimentally proven [13–15], manufacturing optical negative index metamaterials for such a lens for the visible regime still presents significant challenges of fabrication quality and control of losses. A conventional glass lens (as shown in figure 2(c)) also images a spot from the far-field, but the imaged spot is limited by diffraction to about the wavelength of the light used, as evanescent fields are not imaged by a conventional lens.

Below we will show that a lens which is based on the principle of super-oscillation (figure 2(d)) can image a spot into a spot with sub-wavelength resolution without use of evanescent fields. In fact there is no physical limit to the size of the imaged hot-spot created by the super-oscillatory lens. However, we will also show that here the imaged spot is often surrounded by sidebands and the throughput efficiency of
Figure 2. Schematics of different imaging methods, imaging a point source. (a) SNOM. A scanning tip is moved over the surface of a sample, collecting propagating and evanescent waves. The resolution is determined by the size of the aperture in the tip. (b) A negative index superlens. The lens ‘cancels out’ the propagation in free space leading to transfer of evanescent fields from the object to the image and the formation of a perfect image. (c) A conventional imaging lens. The spot size is limited by diffraction. (d) A super-oscillatory imaging lens. The imaged spot is smaller than the diffraction limit, but is often surrounded by sidebands.

Figure 3. Complementary concepts of super-directivity and super-oscillation. On the left a super-directive antenna array generates a narrow super-directive beam that is beyond the diffraction limit. On the right a super-oscillatory lens generates a sub-diffraction hot-spot.

The super-oscillatory lens dramatically reduces with increased resolution. Below we will show how the super-oscillatory lens can be used for imaging by mitigating the problems of sidebands and diminished throughput [16].

2. Discovery of super-oscillation

In 1952, Torraldo di Francia suggested [17] that concepts of super-directive antennas, which were well known in the microwave community [18–20], could be applied to optical instruments to increase their resolution beyond the diffraction limit. The idea of super-directive antennas is illustrated in the left hand side of figure 3. A super-directive antenna is an antenna array of limited size that can direct radiation into an arbitrarily narrow beam by precisely tailoring the interference of waves emitted by different elements of the array [21, 22]. Super-oscillation is a complementary concept where a source of super-oscillatory fields (often a mask) can form, by precise interference, an arbitrarily small spot at a fixed distance from the mask (figure 3, right hand side). In both cases the power delivered to the target beam or spot is only a fraction of the total power input to the system. In super-directivity, the trade-off that has to be made is in the unwanted evanescent waves generated by the input feed in the proximity of the antenna, which consume most of the input power. In super-oscillation we see the unwanted sidebands...
that cannot be fully removed and are formed around the sub-wavelength focal spot.

In 1985, Bucklew and Saleh [23] showed that an ideal, bandlimited, imaging system could be used to image one-dimensional binary images with arbitrary resolution, at the expense of energy losses, showing many of the same features as the phenomenon now known today as super-oscillation. Systematic study of optical super-oscillations was recently revived in the context of quantum mechanics after Aharonov [24] showed that weak quantum mechanical measurements can have values outside the spectrum of the corresponding operator. In other words, a ‘local’ measurement of a value, such as the wavenumber of an optical wave, can be outside the range seen when a global measurement is taken [25].

In 2006, Berry and Popescu [26] showed that this implied optical waves could form arbitrarily small spatial energy localizations that propagate far from a source, and without the need for evanescent waves. However, only a small fraction of energy of the electromagnetic field can exist in the form of a super-oscillation. Ferreira and Kempf showed [27] that the energy that can be channelled into the super-oscillatory region decreases exponentially with the number of super-oscillations. This is a particularly important and prohibiting limitation if one would like to use super-oscillations for communication of high-frequency information using low-bandwidth communication line. The energy channeled into the super-oscillatory region also increases with the speed of oscillation, but only polynomially. In optics this means that as the size of an optical hot-spot is reduced, polynomially less energy of the wave can be concentrated in the hot-spot. The latter fact is crucial for optical super-oscillation to be applied for sub-wavelength light localization and imaging. In principle, there are no limitations on the size of the hot-spot: it can be as small as we wish, providing we do not care how much energy it contains. However, it is the energy of the hot-spot and noise characteristics of the device that determines the practical limits of how small the hot-spot can be while remaining useful in imaging or light-localization applications such as manufacturing with light or data recording.

3. Examples of super-oscillation

While super-oscillation may sound like an exotic and complicated phenomenon, they are often present in complex field patterns but remain unnoticed due to their small amplitudes. For instance, following the approach of [16], it can be demonstrated simply in a one-dimensional wave consisting of six spatial Fourier components:

\[ f(x) = Ae^{i\phi} = \sum_{n=0}^{5} (a_n e^{i2\pi nx}) \]

where \( a_n \) are the Fourier coefficients (here we use \( a_0 = 19.0123, a_1 = -2.7348, a_2 = -15.7629, a_3 = -17.9047, a_4 = -1.0000, a_5 = 18.4910 \)). In figure 4(a) we show the intensity of the wave \(|f(x)|^2\) and its fastest Fourier component \(\cos(k_{\text{max}}x)\) with \(k_{\text{max}} = 10\pi\). The function appears to oscillate slowly compared with the fastest Fourier component, but if we examine the low intensity region near \(x = 0\), as in the lower panel of figure 4, then we see that there is an extremely narrow peak, about 10 times narrower than the fastest Fourier component!

This peak demonstrates two of the characteristics of super-oscillation, firstly that super-oscillatory features are generally of low intensity, and secondly that the phase, \(\phi\), is rapidly oscillating in the super-oscillatory region (red dotted line in figure 4). The rapidly oscillating phase is often used as a signature of super-oscillation [28, 29]: a function is said to be super-oscillatory in a region where the phase gradient \(k_{\text{local}} = \frac{d\phi}{dx}\) is greater than that of the fastest Fourier component of said function, \(k_{\text{local}} > k_{\text{max}}\). That is, the optical field is super-oscillatory if the local wavenumber is greater than the maximum wavenumber present in the wave. This super-oscillatory region is shaded in grey in the lower panel of figure 4.

When characterizing super-oscillatory spots, there are a number of important parameters, illustrated in figure 5. The ratio of hot-spot and sideband intensities is a measure of how efficient the super-oscillatory lens is at collecting light into
Figure 5. A super-oscillatory hot-spot. The hot-spot width is the crucial parameter in determining the resolution of super-oscillatory imaging. The field of view and background ‘grass level’ determine practicality of the super-oscillatory lens in a specific application.

Figure 6. Two-dimensional super-oscillation. (a) Typical intensity distribution of optical super-oscillatory hot-spot (centre of the image) and sidebands (ring surrounding the hot-spot). (b) Corresponding map of phase of the super-oscillatory spot showing rapid changes in the low intensity regions around the spot (red and blue areas correspond to positive and negative values of the phase). (c) Corresponding map of magnitudes of local k-vector (colour reflects magnitude of the local k-vector in units of \(k_{\text{max}}\)). A ring of high local k-vector (\(\sim 3k_{\text{max}}\)) surrounds the sub-wavelength hot-spot.

the small spot. This parameter is crucial in applications such as optical writing in a photoresist or imaging of a live cell. The field of view, i.e. the area around the hot-spot which is not illuminated by sidebands, is also crucially important, as in some applications sidebands may encounter the object of interest, hampering the performance of the imaging apparatus or fabrication tool. The field of view, defined as the area of low intensity, is also characterized by the residual intensity level that is commonly referred to as the background ‘grass level’. The ratio of the peak hot-spot intensity to the grass level intensity determines the finesse of optical fabrication and the noise level of imaging instruments.

We will now look at a more complex two-dimensional field map similar to that generated by binary masks that will be discussed below in section 6. Figure 6 shows the intensity pattern (a) and the phase (b) of a super-oscillatory hot-spot with cylindrical symmetry. In the centre, you can see the focal spot (FWHM 0.44\(\lambda\)) surrounded by annular sidebands. In the phase plot, as in the 1d case, the phase oscillates rapidly around the first null of intensity. This is clearly seen in figure 6(c) where a ring with local wavenumber about three times \(k_{\text{max}}\) is seen around the central spot. The hot-spot is being ‘squeezed’ by the super-oscillatory region into a spot smaller than can be achieved with a conventional lens.

Super-oscillations are also remarkably common in random systems. It was shown in [29] that on average one third of the area of a two-dimension speckle pattern was super-oscillatory and this was extended to waves in more dimensions in [28], although these super-oscillations do not necessarily lead to the small features like those seen in figures 4 and 6. An example of this is presented in figure 7, where a quasi-random speckle pattern is created by a super-oscillatory region that point in random directions in the plane. The intensity of the pattern is shown in (a) and (b), on different scales, and the phase and values of local k-vector are shown in (c) and (d) respectively. As in the one-dimensional case, there are regions of low intensity in which the phase is oscillating rapidly, and in these regions the local k-vector can be many times the maximum k-vector of the input waves. In fact, regions can be found where there are phase discontinuities and the local k-vector diverges [29].

In 2006, Berry and Popescu [26] showed theoretically that super-oscillations could be created by a properly designed diffraction grating, and that they could propagate a significant distance from that grating. Their demonstration considered the evolution of the particular super-oscillatory function

\[
f(x) = (\cos x + ia\sin x)^N \quad (a > 1, \ N \gg 1)
\]

both as a quantum mechanical wavefunction and an optical field in the paraxial approximation. They showed that as the optical field propagates away from a grating that created it, the super-oscillatory sub-wavelength features are maintained over much greater distances than normal evanescent features. Figure 8 demonstrates this propagation; in figure 8(a), a super-oscillatory field is created at \(z = 0\) with parameters \(a = 4, N = 10, \lambda = 0.16\), where \(\lambda\) denotes the wavelength. For these parameters, the shortest spatial scale in the Fourier series is \(2\lambda\), but the finest super-oscillatory structure is of
Figure 7. Super-oscillations are everywhere: two-dimensional optical speckles. (a) Intensity of a random speckle pattern. (b) Zoom of intensity pattern showing super-oscillatory region (white contour) and low intensity region (red contour). (c) Phase of area in (b). The super-oscillatory region is shown by the bright areas. Colour shows phase in radians. (d) Local wavenumber of area in (b) in units of \( k_0 \). The white contours in (b)–(d) are at \( k_{\text{local}} = k_0 \). Figure after [29].

Figure 8. Evolution of super-oscillations. Plots of \( \log |\text{Re}(f(x))| \) for \( a = 4, N = 10, \lambda = 0.16 \), (a) \( z = 0 \), (b) \( z = 1/32 \), (c) \( z = 0.213 \) (super-oscillations expand to \( \lambda \)), (d) \( z = 2.249 \) (disappearance of super-oscillations). Full curve: paraxial approximation, dashed curve: exact solution. Bars show the wavelength \( \lambda \). Figure reproduced with permission from [26]. © 2006 IOP Publishing.

For this wave we are interested in the distance over which the super-oscillations maintain their fine structure. At \( z = 0.213 \), (figure 8(c)) the super-oscillatory features have expanded from their initial size of \( \lambda/4 \) to a size of \( \lambda \), and by \( z = 2.249 \) (figure 8(d)) the super-oscillatory features have grown to size of \( 2\lambda \) and so have effectively disappeared. This shows that, while diffraction causes the super-oscillations to become larger, they can maintain their super-oscillatory
Figure 9. Constructing a super-oscillatory spot. (a) A target spot (dotted black line) is approximated by a super-position of 2 (red), 6 (blue), 10 (pink) and 26 (green) prolate spheroidal wavefunctions. A diffraction limited spot is indicated by the black dashed line. (b) Amplitude (blue) and phase (red) mask of super-oscillatory lens that will create the super-oscillatory spot. (c) Zoom at areas shown by the arrows in (b) showing the phase jumps required in the mask. Figure adapted with permission from [31]. © 2009 American Chemical Society.

nature over a distance of up to $2.249/0.16 = 14$ wavelengths. This demonstration was extended to non-paraxial wave propagation in [30].

4. Designing an optical super-oscillation

As well as the abundance of naturally occurring super-oscillations, it is also possible to instructively design a generator of optical field with particular super-oscillatory features, such as a hot-spot of a particular profile. The simplest way of designing such a generator is to start from the required super-oscillatory hot-spot, then propagate the light backward to a plane and to see what field distribution would be required at the plane. In principle the required field distribution can then be created from a plane wavefront by an appropriate intensity and phase mask. Therefore such a mask can be used as a super-oscillatory hot-spot generator.

In order to design a mask for the sub-wavelength spot, while retaining the bandlimited character of the spot, it is necessary to take account of not only the focal spot, but the sidebands as well, since a small spot in complete isolation cannot be created in the far-field via super-oscillation. To do this, the spot is specified within a given field of view; outside this field of view (that is in the sideband region) the intensity is allowed to take any value.

Indeed, it has been shown in [31] that any arbitrarily small field feature can be represented as a series of bandlimited functions if we are concerned only with a prescribed field of view. This may be achieved using the formulism of prolate spheroidal wavefunctions developed by Slepian and Pollack [32, 33]. This is a complete set of functions which are orthogonal both within the limited field of view and across all space (including any sidebands). The main feature of prolate spheroidal wavefunctions is that they are bandlimited and therefore can be formed from conventional plane propagating waves. Therefore, the mask design algorithm comprises the following steps: first, the desired sub-wavelength hot-spot is represented as a sum of prolate spheroidal wavefunctions, which can be truncated when a satisfactory level of approximation is achieved; second, this sum of prolate spheroidal wavefunctions is decomposed into plane waves, and, using the angular spectrum description of light propagation between the mask to the super-oscillating feature, the required complex mask transmission function $t(x)$ can be readily derived (figure 9).

Here the super-oscillatory hot-spot generator is a mask with spatially varying absorption and retardance that imposes the correct amplitude and phase on an incident plane wave. For an example of the spot and required mask, taken from [31], see figure 9. Fabrication of such masks is a substantial technological challenge. Although it has recently been demonstrated that planar metamaterials can achieve the necessary control of phase and intensity [34], it is certainly desirable to find ways of creating super-oscillatory spots using simpler to manufacture super-oscillatory generators. Below we will explore optical super-oscillations created by binary transmission masks.

5. First observation of optical super-oscillations: quasi-crystal nanohole arrays

The first experimental observations of super-oscillations in optical fields were reported in 2007. They were seen in the diffraction patterns created by quasi-crystalline nanohole arrays [35, 36]. The choice of quasi-periodic masks is not unexpected as, in spite of the simplicity of their design, their Fourier spectrum is continuous and therefore diffraction of light from such masks generates very complex field patterns where one can expect to observe super-oscillatory features, as illustrated in figure 7. Indeed, figures 10(a), (b) and (d) show simulations which demonstrate the formation of sub-wavelength spots in the diffraction from a 5-fold-symmetric quasi-crystalline hole array. Figure 10(a) shows a fragment of the hole array, which has an overall area of $50 \mu m \times 50 \mu m$. Here the quasi-crystalline hole array [37] is a binary transmission mask where white areas transmit incident light, and the black areas are opaque. The rich reciprocal lattice of the array is seen in figure 10(b), and the diffraction pattern
Figure 10. Observation of optical super-oscillations. (a) Fragment of a 5-fold-symmetric quasi-crystalline nanohole array in a metal film. (b) Reciprocal lattice of the array in (a) showing the rich structure in the pattern. Colour scale shows logarithm of intensity in arbitrary units. (c) Sample experimental diffraction pattern from a quasi-crystalline array at 7 µm from the mask when illuminated with white light [35]. (d) Simulated slice through the centre of diffraction pattern created by the nanohole array (a) showing how the pattern changes with propagation (λ = 640 nm). (e) Setup and (f) results of SNOM measurements of sub-wavelength spots created by the quasi-crystalline nanohole array at a distance 6.3 µm from the array, as presented in [36]. Reproduced with permission. © 2007 American Institute of Physics.

7 µm from the array when illuminated with white light is shown in figure 10(c).

Figure 10(d) shows the evolution of the diffraction pattern (for a wavelength of 640 nm) as it propagates away from the array in the z direction. As we move away from the array, complex diffraction patterns are seen that retain some of essential symmetries of the hole array thus representing a complex form of the Talbot effect [35, 36, 38].
In figure 10(e) we show the experimental setup used in [36] for measuring sub-wavelength spots from a quasi-crystalline array. The corresponding experimental results (figure 10(f)) demonstrate how the hole array concentrates light into patterns that retain the 5-fold symmetry of the array. In [36], spots FWHMs small as 235 nm were measured using 660 nm illumination.

Earlier, we mentioned that super-oscillatory lenses could be used not only as focusing devices but also for imaging. Indeed, a quasi-crystalline nanohole array was shown in [39] to be capable of direct point-to-point imaging. A schematic of the experiment is shown in figure 11. A SNOM tip (tapered optical fibre with a sub-wavelength aperture at the end) emitting light at 635 nm was used as a point-like source. The array acted as a super-oscillatory lens to image the point-like source to the other side of the lens. The image was observed using a conventional optical microscope. Indeed, the point source is imaged into an isolated hot-spot, and when the source is moved, the image moves progressively in the opposite direction by the same amount, as would be appropriate for a conventional glass lens with unitary magnification. As the quasi-crystalline nanohole array provides multiple foci at different distances from the array, it can also be used as lens with different magnification without moving the array with respect to the object. In addition, it gives multiple images of the same object on the same focal plane [41]. A noticeable disadvantage of this imaging device is its low throughput efficiency. aberrations also have not been studied, although it has been shown that it can image a relatively complex object consisting of a number of points [40].

6. Binary super-oscillatory lenses

Following the first work on quasi-crystalline hole arrays as focusing and imaging masks, various types of binary super-oscillatory masks have been investigated (figure 12). These are quasi-periodic (figure 12(a)) and quasi-random (figure 12(b)) masks with high-order rotational symmetry and a concentric ring mask with cylindrical symmetry (figure 12(c)). The 27-fold-symmetric quasi-crystalline array in figure 12(a) is produced using the projection method, where the positions of the holes are determined by projecting a 27-dimensional hypercubic lattice onto a plane [42, 43]. By forcing the projection plane to pass through a point in the hypercubic lattice, the array is given a well-defined centre of rotation, and therefore an optical axis on which the focal spots are formed. Savinov also observed super-focusing from semi-random arrays [43], a 40-fold-symmetric example of which is shown in figure 12(b). In order to achieve randomness in the structure while maintaining its rotational symmetry, we place holes randomly on spiral arms around a central point, where the number of arms gives the order of rotational symmetry in the mask. The precise distance between the central point and each hole on a given spiral is randomly chosen, but the holes are at the same position on each of the arms.

More recently, we have also demonstrated focusing from carefully optimized binary ring masks (figure 12(c)) that show much higher throughput efficiency.

The optimization of these ring masks uses an iterative algorithm called binary particle swarm optimization (BPSO), a specific formulation of particle swarm optimization [44–46].
Various super-oscillatory lenses with high-order rotational symmetry and their focal spots. (a) A 27-fold symmetric QNA forming a 0.48λ focal spot. (b) A semi-random spiral hole array forming a 0.39λ focal spot. (c) A ring mask super-oscillatory lens forming a 0.23λ spot in immersion oil. Plate (b) courtesy of Vassili Savinov, University of Southampton.

In this algorithm, schematically illustrated in figure 13, the swarm consists of a certain number of particles \( N \), which move in the \( n \)-dimensional search space to find the global optimum. The position of each particle represents a possible lens design and the quality of the particle at a given position is defined by a merit function—essentially the size of the focal spot. The initial swarm is generated with random starting positions and velocities, representing a random set of SOLs. The focal spot generated by each mask is calculated, and the quality of each spot (and hence each lens) is evaluated. Each particle is moved a small distance (determined by its velocity vector), and its velocity vector is updated to guide its movement towards the global best position achieved by the swarm and its own personal best position. The performance of each particle is then evaluated at its new position, and the cycle is repeated with the aim of the particles converging to the position of the global optimum mask.

An example of mask used by this method is described in [47]. To apply this approach, we divide the radially symmetric ring mask into \( n = 200 \) pixels each 100 nm wide, each of which corresponds to a ring. Each ring can have either unit or zero transmittance—it can be either on or off. The focal spots from each mask design are simulated using the angular spectrum method (see e.g. [48]) assuming plane wave illumination with a wavelength \( \lambda = 640 \) nm. We define the merit function, which we aim to minimize, as the FWHM of the central spot at the focal distance of 10 \( \mu \)m. Furthermore, we require the useable field of view to be 1.2λ and the intensity ratio between the central peak and the maximum sideband to be higher than 0.05. We iterate the algorithm 10 000 times with a swarm of 60 particles.

![Figure 13. Schematic of the particle swarm optimization algorithm for designing a binary ring mask. \( N \) particles swarm around the search space, guided by the best existing positions, converging to an optimum mask design.](image)

It should be noted here that super-oscillatory fields can be experimentally characterized using scanning near-field microscopy as illustrated in section 5. Moreover, as was first noted in [35], as super-oscillatory hot-spots are formed by free-space waves with limited \( k \)-vector, they can be imaged and magnified by conventional lenses, as long as the lens can pick up not only the hot-spot itself, but the entire super-oscillatory field, including high intensity lobes, as they are inseparable parts of the super-oscillation. Although objectives
cannot collect the evanescent waves that can, in principle, be collected by a SNOM probe, super-oscillatory spots are formed in free space and so will contain no significant evanescent components. In practice, super-oscillatory lenses often have foci close to the lens so conventional lenses that can image super-oscillation should have near-unity numerical aperture and large field of view which is often difficult to achieve simultaneously. However, from our experience, numerous experiments show that imaging of super-oscillations with either dry lenses with NA $\geq 0.95$ or oil immersion lenses with NA $\geq 1.4$ gives good agreement between parameters of measured and theoretically calculated hot-spots.

Figures 12(a) and (c) show experimental results obtained by microscope observation (panel (a) in air; panel (c) in immersion oil) of the diffraction pattern from lenses manufactured using e-beam lithography and focused ion beam milling respectively. Figure 12(b) is a simulation result using the angular spectrum method. In all cases the wavelength used is 640 nm. These spots all have different characteristics and may, therefore, be useful in different applications. The binary super-oscillatory lens (SOL) in figure 12(a) produces a sub-wavelength (0.48$\lambda$) spot on a large 90$\lambda$ field of view. The spot is in figure 12(b) is smaller in size, but has sidebands of moderate intensity nearby the spot. In figure 12(c), we have formed a spot that is only 0.23$\lambda$, but has sidebands of around the same intensity as the spot very close by. This spot is still suitable for use in, for example, imaging, as we will demonstrate below, although the presence of sidebands so close to the spot may reduce its utility in other applications.

As well as spot size, the efficiency of focusing and the intensity in the focal spot are also important in determining the usefulness of binary SOLs. The SOLs are somewhat less efficient than a traditional lens because they have opaque regions, while a conventional lens is transparent over its whole aperture, but this loss of efficiency is not extreme. The hole arrays have only about 5–10% of the lens area transparent, but the ring masks have a much greater transmission, at around 50%. Another source of inefficiency is that only a proportion of the light that has passed through the mask is found in the super-oscillatory spot, with the rest in the surrounding sidebands. While this problem is intrinsic to any super-oscillatory focus, it does not prevent their use in imaging applications where the number of available photons available is vastly greater than the detection limit of a good detector. In the case of the spots seen in figure 12, a few per cent of the light is typically found in the focal spots, while the intensity in the spots is between 10 and 100 times that incident on the binary SOL showing that the binary SOLs still achieve a significant concentration of the incident light.

Binary SOLs can be fabricated using two standard nanofabrication processes; focused ion beam (FIB) milling or electron beam lithography (EBL). To manufacture a mask using FIB, a thin metal film is deposited on a transparent substrate using thermal evaporation. The SOLs focus light through interference of the waves transmitted through the mask, the exact film thickness is not critical to mask performance, as long as it is thick enough to be opaque to the light being focused, and we typically use 100 nm thick film. Similarly, it is possible to use a number of metals for the SOL, without affecting lens performance. We have successfully manufactured masks from gold, titanium, chromium and aluminum. In general, harder metals, such as chromium or titanium, take longer to mill in the FIB, but give better fabrication quality. The deposited film is then directly milled in the FIB, removing those parts of the mask that are to transmit light (figure 14(a)).

We also fabricated SOLs using electron beam lithography (EBL). To manufacture an SOL using EBL, we use a lift-off process [49–51] where a thin layer of resist is spun onto the substrate. The resist is patterned by exposure to an electron beam and developed, leaving resist only in the transparent areas of the final SOL. Then a metal film is deposited on the substrate, over the resist. Finally the remaining resist is removed, which also removes the metal on top of it, leaving the required design (figure 14(b)). Figures 14(c)–(f) show a nanohole array and ring mask fabricated by EBL and FIB respectively, showing the quality that can be achieved.

![Figure 14](image-url)
Binary SOLs are a promising technology for achieving super-oscillatory focusing. They can be manufactured relatively easily using existing nanofabrication techniques, and are simple to install in optical systems, requiring minimal alignment. They have also been demonstrated to give sub-diffraction limited performance both in air and in immersion oils. One drawback of the SOLs is the relatively short focal distances that have so far been demonstrated, where the focal length is determined by the radius of the SOL (the smallest spot size is related to the largest wavevector that can contribute to the spot at a given distance from the lens). By manufacturing larger lenses, the focal length can be extended to hundreds of microns, equivalent to a high NA conventional objective.

7. Imaging with super-oscillatory binary masks

Super-oscillatory lenses have been used for imaging in a real microscope system [47]. This system uses the focusing ability of super-oscillatory lenses, combined with a confocal microscope arrangement, to achieve imaging with resolution better than $\lambda/6$. An incoming laser beam (wavelength 640 nm) illuminates a binary ring mask SOL. This SOL focuses the spot into a sub-wavelength spot about 10 $\mu$m from the lens. The sample to be imaged is scanned across the sub-wavelength spot, and the transmitted light is collected by a conventional microscope lens and imaged on a digital camera. From this captured image, only the light from the central region is recorded, corresponding to the region where the focal spot would be if the sample were not

---

**Figure 15.** Imaging with an SOL equipped microscope. (a) Photograph of the super-oscillatory microscope. (b) The focal spot formed 10 $\mu$m from the lens is simulated (b) and measured (c) to have a FWHM of 185 nm. (d) Design and (e) SEM image of the object: a cluster of nanoholes. (g) Simulated confocal image of the object. (j) Simulated image of object using SOL. (f) Spatial frequencies present in (g) on logarithmic colour scale and theoretical limit (dashed line). (i) Spatial frequencies present in (j) showing the considerable intensity outside the (dashed line) limit for a confocal system. (h) and (k) show experimental microscope and SOL images respectively. Panel (l) shows the effect of simulated detector or laser noise on imaging: contrast is reduced, but the resolution remains the same. Figure adapted with permission from [47]. © 2012 Nature Publishing Group.
present. This process recreates a conventional confocal microscope [52–55], but with a super-oscillatory focusing lens, and using an electronic pinhole, rather than a physical aperture. By using this confocal approach, the light collected to form the image is dominated by that scattered from the central sub-wavelength spot. No subsequent computational processing is needed to achieve a super-resolution image.

Simulated results using this imaging system are shown in the left half of figure 15. The focal spot formed 10 μm from the SOL in immersion oil has a FWHM of 185 nm and is shown in figure 15(b). This spot is scanned across the object (figure 15(d)) and confocal imaging (with a 1.4 NA objective) is simulated and shown in (figure 15(j)). For comparison, a conventional confocal image simulated with the same numerical aperture is shown in (g). The improved resolution can be seen by simply comparing (g) and (j) directly, but for a more quantitative measure, we also compare the spatial frequencies present in the images.

The spatial frequencies present in an image give a good measure of the finest detail visible in the image or, equivalently, the image resolution. The object hole cluster shown in (d) has sharp edges, and so will have spatial frequency components at all spatial frequencies. A conventional optical imaging system can only capture spatial frequencies less than \( k_0 \times NA \) and so causes a blurring of the image, as seen in figure 15(h), where sharp edges are lost. For a confocal microscope the spatial frequencies captured are larger by a factor of two, hence the improvement in resolution seen between (h) and (g). If, as expected, the SOL captures a higher resolution image, then that image will contain even higher spatial frequencies.

For our simulated results, we calculate the spatial spectra using the two-dimensional Fourier transform of the images:

\[
F(k_x, k_y) = \frac{1}{2\pi} \int \int f(x, y) e^{-i(k_xx + k_yy)} \, dx \, dy.
\]

In (f) and (i), the power spectra \( |F|^2 \) are shown on a logarithmic colour scale, and the theoretical limit for a confocal microscope \( \sqrt{k_x^2 + k_y^2} \leq k_0 \times 2NA \) is shown by the black dashed line. The extra resolution of the SOL image is signified by the presence of intense components with high values of wavevectors located outside the dashed circle that are not present in the spectrum of the confocal image.

To convert the super-oscillatory imaging concept into a working instrument, we have modified a standard optical microscope to use a super-oscillatory lens (figure 15(a)). The super-oscillatory lens shown in figure 14(d) is used to replace the focusing objective and produces a super-oscillatory spot with FWHM 185 nm (figure 15(c)). We use a nanofabricated test object consisting of a 100 nm thick gold film with the hole cluster milled into it using FIB (figure 15(e)). A conventional microscope image of the object is shown in (h), with the individual holes completely unresolved. However, the SOL can clearly resolve all but the closest sets of holes (figure 15(k)), including those separated by just 105 nm.

Many super-resolution techniques, particularly those which rely on computational improvements in resolution are highly susceptible to noise in the image [4, 56], and only offer resolution improvements if low noise images are used as input. Super-oscillatory imaging, however, is robust against noise, as demonstrated in figure 15(l). Here we see images where various levels of photo-detector noise are applied to the captured images before the electronic pinhole is applied. The noise level is measured as the standard deviation of the noise, normalized to the peak intensity of the hot-spot (η). Even for the relatively high noise levels of \( \eta = 8\% \), a clear pattern of the test object is seen and the resolution is not reduced, although the contrast goes down, as would be expected in any noisy detection system.

8. Dynamic super-oscillatory lenses with spatial light modulators

Although binary masks provide robust and simple to implement sub-wavelength hot-spot generators, super-oscillatory focusing can also be achieved using spatial light modulators (SLM) and a conventional microscope objective that focuses a beam with a carefully designed amplitude and phase profile for precise tailored interference. Conceptually, this is similar to works on super-focusing with pupil filters, as first discussed by Di Francia [17], and developed in, for example, [53, 55, 57] and reviewed in [4]. Such precise programmable shaping of the beam may be achieved when the phase and/or amplitude modulating surface is imaged onto the back focal plane of the objective (figure 16). An input beam from a fibre-coupled laser is expanded and collimated, and is incident on an amplitude shaping SLM. The amplitude modulated beam is then imaged onto a phase modulating SLM, using a telescope. This phase and amplitude spatially modulated beam is then demagnified and imaged onto the back focal plane of the microscope objective: the field on SLMs and the focal plane of the objective are then related by a Fourier transform. In this setup any arbitrary field profile may be obtained at the microscope focal plane simply by encoding its Fourier transform on the SLMs.

To achieve super-oscillatory focusing in this way, we need to determine the amplitude and phase pattern incident on the objective that will create the desired focal spot. The optical eigenmode method [58, 59] is well suited to this problem. In this process, described fully in [58], a series of test masks are displayed on the SLMs to probe the optical system (lenses, free space, and microscope objective) between the SLM and the desired focal plane. This probing allows the intensity and spot size matrix operators of the system to be determined. The eigenvectors of these operators give the optical eigenmodes of the system, and the eigenvalues give the intensity and spot size of the respective eigenmode. It has been shown in [58] that the optical eigenmode mode with smallest spot size (defined as second-order moment), for a given field of view, produces the smallest spot achievable, given the test masks and the optical system used. In this way the SLM pattern that forms the smallest spot can be determined without iterative optimization of the system, and the method automatically takes account of all aberrations present in the system. This optical eigenmode optimization is not limited to optimizing super-oscillatory spots, and has recently been used to optimize patterns of light
Figure 16. Creating super-oscillations with spatial light modulators. Schematic (top) and photograph (bottom) of the setup for super-oscillatory focusing with SLMs. Light from a fibre-coupled laser is collimated, expanded, polarized and is incident on an amplitude modulating SLM. This beam from this SLM is imaged onto the phase SLM with a 1:1 telescope, and then demagnified onto the black focal plane of a microscope objective using a further telescope system.

Figure 17. Super-oscillatory focusing using SLMs. (a) Outline of the experimental optimization process. Step 1: extract intensity cross-section (rhs) from measured 2d intensity of BBs (lhs). Step 2: fit Bessel function to 1d cross-sections. The black curve is the fitted intensity cross-section. Step 3: optical eigenmode optimization. The horizontal red lines indicate a range of interest as used for the optimization procedure. Step 4: implementation and characterization of the final optimized mask. (b) 2d intensity map (top) and 1d intensity cross-section of measured optimized spot. Dots: measured intensity. Continuous curve: predicted intensity. (c) Intensity cross-section of theoretical super-oscillatory spot showing super-oscillatory region (black-filled area). Figure adapted with permission from [62]. © 2011 American Institute of Physics.
in nanoscale landscapes [60] and for indirect imaging of an object [61].

This method has experimentally demonstrated to achieve a 222 nm spot at a wavelength of 633 nm: a size of 0.35λ and 45% of the width of the corresponding diffraction limited spot [62]. In this implementation, summarized in figure 17, a series of 15 Bessel beams were used to probe the system. Each Bessel beam is formed by displaying an annulus of given size on the SLM (the test masks), forming a beam with a Bessel beam intensity profile \( I(x) = J_0(x/x_0) \) where \( J_0 \) denotes the zero-order Bessel function, and \( x_0 \) is a scaling factor, which is different for each Bessel beam. These beams are measured using a collection mode SNOM tip, as shown in figure 10, scanning over the focal plane of the microscope objective. Each measured beam profile is fitted to an ideal Bessel beam profile and these fitted functions are used as input to the eigenvmode optimization algorithm. The output is a mask for display on the SLM, consisting of a sum of the input test masks, each multiplied by a (complex) amplitude. When this mask is displayed on the SLM, the super-oscillatory spot is generated and is measured with the same scanning detection scheme. The experimental results, together with the theoretically predicted spot, are shown in figure 17(b). To demonstrate the super-oscillatory nature of the spot, the local wavenumber is calculated for the theoretical spot, and the super-oscillatory region is denoted by the black-filled area in figure 17(c).

This optical eigenmode approach was also used to achieve sub-diffraction limited imaging resolution by Kosmeier et al [63] in 2011. In this experiment, they compare confocal imaging (similar to the binary mask arrangement used in section 7) in a low NA arrangement using a conventional lens on its own, and a conventional lens combined with an SLM to produce a super-oscillatory focal spot, using a similar method to [62]. The results are shown in figure 18, where they achieve a resolution of 65 μm, a factor of 1.3 improvement over the 82 μm theoretical diffraction limit for their system.

SLM super-oscillatory focusing, while similar in principle to focusing with binary SOLs, has a number of experimental differences: The SLM approach allows focal spots to be formed at the focus of a microscope objective, giving a working distance of at least a few hundred microns, compared to the few tens of microns currently seen with binary SOLs, making experiments on rough surfaces or within thick objects easier. SLM focusing is also more suitable for reflection, or epi-fluorescence, imaging, as it does not require a double pass through a binary SOL, but only through a standard microscope objective, and it may also be more suitable for optical scanning of the super-oscillatory spot as performed in a laser scanning confocal microscope. On the other hand, the SLM system is more complex to align, consisting of a number of optical components and considerable free-space path length. The binary SOLs have only one component and require only a relatively easy to achieve collimated laser beam as input. The SLM system is also sensitive to aberrations in the optical system. Although these are somewhat corrected for by the eigenmode optimization, this does not remove all aberrations. Further improvements may be made using correction procedures such as those described in [64], but this adds another complexity to the control system. Both binary SOLs and SLM super-oscillatory focusing rely on the spatial coherence of the input beam. In [62], it was shown that a partially coherent beam is focused into larger hotspots than a coherent beam, but
also that the degradation can be partially corrected by taking the coherence of the beam into account when designing the focusing mask.

Super-oscillatory optical imaging has also been achieved by placing an SLM in the collection path of a microscope, to optically process the captured light and increase the resolution of the imaging system [65]. This creates a microscope with a simpler setup, and does not require scanning of the beam over the sample, but also does not suppress the sidebands created by the filter, restricting it to operate within the field of view of the super-oscillatory spot. A similar approach has been used to for computational super-oscillatory filtering of images in [66].

9. Limitations of super-oscillatory imaging

Although super-oscillations—in principle—have no physical constraints on the size of spot that can be created, the super-oscillatory imaging implementations discussed above do have some limitations.

The first limitation is in the manufacturing of the SOLs themselves. Ideal SOLs of arbitrary design can be achieved by masks with continuously variable retardation and optical density (see section 4). These masks must have nanoscale accuracy and very high density and retardation tolerance to work in the visible part of the spectrum. However, currently no fabrication technique can deliver such continuous masks. Although dynamic masks created with spatial light modulators offer good control of intensity and phase of the optical field [58, 62, 63], mask pixilation of the current SLM technology at the level of about 10 µm is a limiting factor, as is the flatness of the SLMs and aberrations in the necessarily complex optical system. The development of continuous SLMs with much smaller pixilation should make good dynamic masks possible. If a recently demonstrated reconfigurable photonic metamaterial with sub-wavelength elements [67] could be made randomly addressable, this would be a solution for achieving super-oscillatory masks by controlling retardation and intensity through scattering of light on metamolecules forming the metamaterial mask [34] and could work without the need for complex optical arrangements. At present, binary mask technology remains the best practical option. However it also faces limitations due to fabrication, in particular for short wavelength application and smaller spots: as smaller spots are created by more precise interference, the tolerance to scattered light, from fabrication imperfections and inaccuracy, reduces. The implication of practical limits of currently available nanofabrication technologies on the performance of binary mask technology remains to be investigated.

The second limitation is in scattering from the sidebands around the hot-spot spot. As the spot size is reduced, the sideband intensity relative to the spot intensity always increases. This increased intensity creates more scattering. For instance, in the imaging apparatus described on figure 13 scattering transmits some intensity through the pinhole and forms some distortions in the image. This problem can be illustrated by simulation presented in figure 19: when a low intensity super-oscillatory spot (figure 19(a)) with a FWHM of 0.16λ is used to image a point source (this is similar to measuring the point spread function in standard microscopy), the image will have noticeable sidebands around the spot (figure 19(b)). Here simulations are carried out assuming conditions of the experiment described in [47].

To investigate the effect of spot size on imaging performance, we need a way of constructing super-oscillatory spots of arbitrary size, while keeping them bandlimited. We do this using a variation of the method described in [68], where the authors show that a super-oscillatory function can be created by shifting a finite number of the zeros of any known bandlimited function. Consider the one-dimensional case of the function \( f(x) = \sin c(\pi x) \equiv \sin(\pi x)/\pi x \), which is bandlimited and is zero for any integer \( x \). If we take this sinc function and distort it slightly, by moving its zeros, we can make arbitrarily small features in a certain field of view, without increasing the bandwidth of the function [68]. To construct a super-oscillatory spot, the innermost zeros, at \( x = \pm 1 \), are moved towards zero, to squeeze the central spot to a smaller size.

To make two-dimensional spots using this method, we use the Airy disc function instead of the sinc function as the initial function, and shift the zeros in the radial coordinate, \( r \). The Airy disc is defined as \( I_{\text{Airy}} = |2J_1(r)/r|^2 \) where \( J_1 \) is the first-order Bessel function of the first kind. To move the zeros mathematically, we modify the Airy disc as:

\[
I(r) = \left| \left( \frac{2J_1(r)}{r} \right) \prod_{i=1}^{n} \frac{r^2 - a_i^2}{r^2 - z_i^2} \right|^2
\]

where \( z_i \) are the zeros of \( J_1(r) \) and \( a_i \) are the new zeros of the function. To create spots with particular parameters, a multi-objective genetic algorithm [69] was used to find the zeros \( a_i \) that gave the spots with the lowest sideband intensity for a given spot size.

Figures 17(c) and (d) illustrate how within this model the rings formed by scattering of the sidebands change with reducing spot size. They present central cross-sections of the super-oscillatory hot-spot and its point image as the function of hot-spot size (vertical axis). As the super-oscillatory spot size reduces below 0.25λ noticeable sidebands appear around it (c), becoming more intense with further reduction of the hot-spot’s size. However, in the image formed using that spot (d) the sidebands are much less intense, becoming significant only when the hot-spot size drops below about 0.15λ.

These simulations show that the setup in section 7 with optimally designed SOLs could give resolution of 0.15λ, but that extending the resolution any further cannot be achieved simply by making a smaller spot. It will require either a using a super-oscillatory spot with larger dark field of view before the first sideband, or a different method of suppressing the sidebands in the image, perhaps by making the polarization of the sidebands different to that of the spot [70].

10. Super-oscillatory imaging within the field of view of the super-oscillatory hot-spot generator

The above given examples are concerned with imaging of objects that are much bigger than the field of view. Here,
confocal detection is necessary to suppress scattering of light from the side lobes to achieve a $\lambda/6$ resolution and reasonable level of image distortion.

Remarkable opportunities open for high resolution imaging if the object is smaller than the field of view of the hot-spot of the super-oscillatory lens. (The region around the central spot in which there are no sidebands of significant intensity; see figure 5.) In this case we shall only deal with low intensity scattering from the ‘field of view’ area near the hot-spot, and should not worry about scattering from the high intensity sidebands, as the object fits within the field of view: image distortion is reduced substantially.

To illustrate this we simulate hot-spots with sizes of $\lambda/5$, $\lambda/9$, $\lambda/21$ corresponding to FWHM of 80, 43 and 19 nm at the wavelength of 400 nm. The generated hot-spots have field of view full widths of 397 nm, 186 nm and 76 nm ($0.99\lambda$, $0.46\lambda$, and $0.19\lambda$) respectively. In figure 20 one can see that such super-oscillatory lenses can resolve two holes spaced by 113 nm, 63 nm and 28 nm respectively, while standard confocal microscopy fails completely. The latter one would correspond to resolution up to $\lambda/14$.

Figure 19. Modelling limits of imaging with a super-oscillatory hot-spot. (a) Super-oscillatory spot with FHWM 0.16$\lambda$ with intense sidebands and (b) confocal image of a point using an SOL generated spot (a). (c) and (d) Series of lineouts through white dashed lines in (a) and (b) for a series of spots with varying sizes.

However, this resolution would be very difficult to achieve experimentally. Indeed, in this simulation the super-oscillatory spots are a factor of $10^7$, $10^{13}$ and $10^{19}$ weaker, respectively, than the sidebands surrounding them. It is not the low hot-spot intensity that represents the main challenge: a 1 W laser delivers more than $10^{18}$ photons s$^{-1}$ and a high quantum efficiency detector can be employed. It is the stray scattered light from imperfections of the optical system that will be the main problem: this will spoil the image unless it is not suppressed to the same level as light coming from the hot-spot itself. This may require working in vacuum to minimize air fluctuations and engagement of sophisticated techniques to minimize scattering.

However, more can be done regarding design of the hot-spot generator itself: in the above examples the hot-spots were optimized for spot size and field of view only. The remaining challenge is to investigate how lowering the ‘grass level’ requirement (see figure 5) or accepting higher image background noise could be used to increase intensity of the hot-spot by changing the SOL design. What also remains to be explored is how nonlinear optical imaging techniques such as multi-photon and CARS imaging could be used with SOL
Figure 20. Super-oscillatory imaging within the field of view of an SOL. First column: super-oscillatory spot of varying sizes with relatively large fields of view ($\lambda = 400$ nm). Second column: objects, consisting of a pair of 9 nm nanoholes with varying spacing. Third column: simulated confocal images of the objects showing no resolution. Fourth column: simulated SOL images of the objects showing that they are clearly resolvable. In each row, the scale is the same in the last three columns.

type light concentration devices. With these challenges ahead, the above computational results demonstrate a remarkable potential for imaging of truly nanoscale structures.

11. Non-imaging applications of optical super-oscillations

Having looked at imaging applications, we will now explore other directions of research in the field. The first direction goes back to the idea of making a super-oscillatory lens from a mask with continuously variable amplitude and phase. While such a lens is difficult to fabricate using traditional technologies, planar metamaterials (flat materials with sub-wavelength structuring) have recently been shown to allow control over the amplitude and phase of transmitted or reflected light [71–73]. As the units that make up the metamaterial are fundamentally sub-wavelength, this technology should allow the realization of the easy-to-design amplitude and phase masks. A simple example of a meta-lens capable of super-oscillatory focusing has recently been demonstrated [34].

SOLs also could provide important opportunities or data-storage applications. Magnetic hard disks are a vital technology in modern computers and rapid technological improvements have historically led to an approximate doubling of disk density every two years. Continuation of further progress in these areas however, is now under threat, as magnetic hard drives approach the superparamagnetic limit [75]. Heat assisted magnetic recording (HAMR), in which the magnetic material is locally heated during the magnetic writing process, is one of the leading industry road-map technologies currently under development to beat this limit. The preferred technology for heating the magnetic platter is focusing infra-red diode lasers using tapered plasmonic waveguides. These waveguides are inefficient as well as being difficult to manufacture. A recently developed type of SOL, the optical needle SOL (ONSOL) [74] may provide an alternative solution. These ONSOLs focus light into sub-wavelength needle, rather than a spot. Moreover the needle is well separated from any sidebands of significant intensity—giving the optical needle a large field of view in which to work. These ONSOLs are designed in the same way as SOLs, but they have one crucial difference: the central area of the ONSOL is always opaque, blocking the incident light and forming a shadow region in which the optical needle is formed. The size of the blocking region determines the length of the optical needle and the distance between the needle and its sidebands [74]. An example of such an ONSOL and the optical needle it forms are shown in figures 21(a) and (b). For a wavelength of 640 nm, a 7 $\mu$m long needle with width of 0.4$\lambda$ is formed starting 4 $\mu$m from the lens.
Other techniques for producing optical needles by modulating a radially polarized beam incident on a high NA lens have also been proposed [77, 78]. Non-diffracting super-oscillatory needles were also demonstrated in [79], but all these other needles require conventional lenses and are not as easily integrated into the micron scale HAMR apparatus.

In [74], the size of the needle was only around 0.4\(\lambda\), or 268 nm, but HAMR applications require a spot around 40 nm. To push the needle performance towards that goal, we move to a shorter wavelength laser, at 473 nm, where diodes are commercially available, and also employ a solid immersion medium in which the spot is formed, as shown in figure 21(c). We simulate focusing using an ONSOL with dark central region of 6 \(\mu\)m radius (figure 21(d)) in a GaP immersion medium of refractive index \(n_{\text{GaP}} = 3.7 + 0.01i\) [80]. For a 4.7 \(\mu\)m thick immersion medium, a focal spot with a FWHM of 51 nm is formed at the exit face of the medium, as seen in figures 21(e)–(f). This spot is suitable for heating of the magnetic disk in HAMR. The coupling of this spot into the magnetic disk is complex and requires investigation, but we do not anticipate this causing a significant problem. This spot is separated from the nearest intense sideband by more than 6 \(\mu\)m—a separation large enough to allow realistic use of the spot in HAMR.

As well as storage applications in HAMR, ONSOLs may be used for readout of optical memory with sub-wavelength pitch. Here figure 22 illustrates reading of an optical memory track with pitch \(\lambda/4\) consisting of the 4 byte sequence [00111001 00111100 11001010 00011101]. The sample was manufactured as a series of FIB milled slits in a 100 nm thick gold film on a silica cover-slip. Each slit was 320 nm \(\times\) 160 nm spaced with a 160 nm pitch (figure 22(b)). We use the confocal-like immersion imaging system described in section 7 with the ONSOL in figure 22(a) for illumination and an oil immersion 1.4 NA objective (Nikon VC100 \(\times\) H) for imaging. The wavelength is 640 nm. Confocal images using a conventional objective lens (Nikon NCG100 \(\times\) H) for focusing (spot FWHM 392 nm) and using the DFSOL (spot FWHM 211 nm) are shown in figures 22(c) and (e) respectively. When the signal from each image is digitized (by taking pixels from a line of the image in the centre of the track and averaging over each 160 nm pit), the conventional objective is unable to read the data stream, giving four errors (red boxes in figure 22(d)). As expected, the read errors occur mainly in those small features which are unresolved by the conventional objective. The ONSOL, on the other hand, is able to read the entire data stream with no errors (figure 22(f)).

12. Super-oscillatory imaging in context

As we have seen, optical super-oscillations offer a promising new technology for super-resolution imaging. However, there are a number of existing super-resolution techniques, both commercially available and proposed in the literature, and here we consider the relative merits of some of these compared to super-oscillatory imaging.
Super-oscillatory imaging provides super-resolution without evanescent waves, and therefore without being in the near-field of the object. This is of vital importance when imaging within structures, such as biological cells, where a probe cannot be brought within the necessary few nanometres of the object. It does not require fluorescent labelling of the object, and it has been demonstrated on non-fluorescent samples but, if fluorescent labelling is advantageous for a particular application, then it is compatible with any label suitable for use with existing confocal microscopes. It can, in principle, image at the same speeds as a confocal microscope, allowing live cell imaging, and it does not rely on any image processing algorithms, meaning it is robust to noise. In contrast, the promising, and commercially available, technique of stimulated emission depletion microscopy [81–83], is more optically complex, in that it requires two beams of different wavelength to be aligned with each other in three dimensions and, while it can operate at high speed, it requires very specific fluorescent properties to be present in the sample [2]. A number of stochastic methods, for example, photoactivated localization microscopy (PALM) [84] and stochastic optical reconstruction microscopy (STORM) [85], are also commercially available and are optically simple, but these are relatively slow, requiring the capture of thousands of standard resolution images to obtain a single high resolution image, restricting their use in live cell imaging. Again, specific fluorescent properties are required within the sample, meaning that only samples labelled with one of a few special dyes can be used.

Other techniques not based on fluorescence, such as SNOM [86], the ‘poormans superlens’ [13], and the hyperlens [87–89] have demonstrated super-resolution imaging of nanostructured systems, but all require the imaging lens or probe to be placed in the near-field of the sample which, as described above, puts a significant restriction on the samples that can be imaged. A true negative index superlens [12] can overcome this near-field requirement, but a low loss, negative index metamaterial in the visible range remains beyond the limits of current fabrication technologies. Other computational techniques, such as sparsity based super-resolution imaging [90–92] and ptychography [93] are in general sensitive to noise in the acquired data or require a significant amount of a priori information about the object [56]. Focusing using disordered media, described in [94, 95] and reviewed in [96], provides another interesting way to increase the focusing power of a lens, although it is still subject to the diffraction limit when used for far field imaging. Super-resolution has been achieved with an object in the near-field of the disordered sample [97], but this again relies on capturing the evanescent components of the scattered light.

Compared to these techniques, super-oscillatory imaging offers a capability for un-labelled, far-field imaging which is robust to noise and optically simple. While challenges remain in the realization and optimization of super-oscillatory instruments, they provide a powerful new and complementary capability to existing techniques. Table 1 summarizes the state of the art in experimental super-oscillatory focusing, comparing the different spots so far observed. For each spot, the wavelength and numerical aperture of the focusing device are used to calculate the diffraction limited spot size $d = \lambda/(2NA)$. The size of the observed spot $w$, relative to the diffraction limit, gives a measure of the level of improvement achieved by super-oscillatory focusing.

13. Conclusions

Recent work in the field of optical super-oscillations has shown great potential for all optical super-resolution imaging, without the need for specific fluorescent properties in the sample. While resolution of $\lambda/6$ has been achieved in fabricated test objects, extension into imaging of objects of scientific importance, either in the life or physical sciences, remains an open challenge. As well applications in imaging, super-oscillatory focusing has applications in
materials processing, for example, and, as discussed above, heat assisted magnetic recording. The continued development of nanofabrication and other technologies, such as beam shaping meta-surfaces, will allow the realistic creation of spots better optimized for specific applications—in imaging and elsewhere. While technological and physical challenges remain, optical super-oscillations clearly provide a paradigm for a new optical super-resolution technology with numerous applications.

Acknowledgments

The authors are grateful to M Mazilu, K Dholakia, S Kosmeier, J Baumgartl, T Vettenburg, M R Dennis, J Lindberg, J Ring, Y Chen, T Roy, S Savo, T-S Kao, J E Chad, V Savinov, C-J Chuang, Y Guanghui, and A D Boardman for numerous fruitful discussions on the physics and applications of super-oscillations. This work was funded by the United Kingdom Engineering and Physical Sciences Research Council; grant number EP/F040644/1, the Royal Society, London, A*STAR, Advanced Optics in Engineering Programme, Singapore, and the Ministry of Education, Singapore, under grant MOE2011-T3-1-005.

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Table 1. Performance of different super-oscillatory focusing methods relative to the diffraction limit.

<table>
<thead>
<tr>
<th>Method</th>
<th>Wavelength (nm)</th>
<th>Numerical aperture</th>
<th>Diffraction limit, d (nm)</th>
<th>Spot FWHM, w (nm)</th>
<th>Relative size (d/w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONSOL [74]</td>
<td>640</td>
<td>0.96</td>
<td>334</td>
<td>268</td>
<td>0.80</td>
</tr>
<tr>
<td>Binary ring masks [47]</td>
<td>640</td>
<td>1.24</td>
<td>257</td>
<td>185</td>
<td>0.72</td>
</tr>
<tr>
<td>Quasi-crystalline nanohole arrays [36]</td>
<td>660</td>
<td>1.00</td>
<td>330</td>
<td>235</td>
<td>0.71</td>
</tr>
<tr>
<td>Binary ring masks</td>
<td>640</td>
<td>1.37</td>
<td>233</td>
<td>145</td>
<td>0.62</td>
</tr>
<tr>
<td>Optical eigenmode [63]</td>
<td>633</td>
<td>4.32 × 10⁻³</td>
<td>73 300</td>
<td>44 700</td>
<td>0.61</td>
</tr>
<tr>
<td>Optical eigenmode [62]</td>
<td>633</td>
<td>0.65</td>
<td>487</td>
<td>222</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Performance of different super-oscillatory focusing methods relative to the diffraction limit.


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