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Citation: Appl. Phys. Lett. 112, 201111 (2018); doi: 10.1063/1.5026649
View online: https://doi.org/10.1063/1.5026649
View Table of Contents: http://aip.scitation.org/toc/apl/112/20
Published by the American Institute of Physics

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Lattice induced strong coupling and line narrowing of split resonances in metamaterials

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(Received 22 February 2018; accepted 30 April 2018; published online 17 May 2018)

Strongly coupled metamaterial resonances typically undergo mode-splitting by which there is an exchange of energy between matter excitations and photons. Here, we report a strong coupling of the lattice mode with the structural eigen-resonances of an asymmetric split-ring metamaterial associated with mode-splitting and resonance line-narrowing that gives rise to high quality factor (Q-factor) resonances. We demonstrate selective control of the resonance strength, line-width, and Q-factor of individual split-ring modes by tailoring the coupling of the fundamental lattice mode to each of the hybridized resonances. A three-coupled-oscillator model shows lattice-mediated strong coupling in the form of an anti-crossing behavior between the hybridized metamaterial resonances. Such schemes of strong coupling between the lattice and the hybrid modes of the metamaterial unit cell offer an avenue to invoke lattice induced transparency, high-Q resonances and strong field confinement, which could find applications in designing slow light devices, ultrasensitive sensors, and multiband narrow filters. Published by AIP Publishing. https://doi.org/10.1063/1.5026649

Metamaterials1,2 are artificial structures engineered to display unique optical properties that are absent in natural materials, such as negative refraction,3,4 invisibility cloaking,5,6 perfect lensing,6,7 sensing functionalities,8 and phase compensation.9 These artificially engineered materials are composed of so-called metamolecules of subwavelength size that are typically designed with high conductivity Drude metals structured in a periodic array. Due to their subwavelength periodicity, they interact homogenously with the incident electromagnetic field and can exhibit strong resonant responses. Earlier studies have shown that different metamaterial resonances can be coupled to achieve narrow resonant features mimicking electromagnetically induced transparency (EIT).10-14 EIT type resonances in metamaterials involve near-field coupling between superradiant “bright modes” and subradiant “dark modes.” Bright modes are highly radiative and directly excited by the incident electric field. While dark modes arises from the asymmetric near-field coupling of the metamaterial elements, and are weakly coupled to the incident field allowing them to trap electromagnetic energy within the resonator.15 Similarly, metamaterial resonances could also be coupled to elementary excitations such as excitons16 or phonons17-19 to observe similar phenomena.

Since metamaterials are periodic structures with subwavelength lattice parameters, another class of dark modes exists in the array, the lattice modes. A lattice mode corresponds to diffraction along the interface of a periodic structure and mimics a surface plasmon polariton wave. Lattice modes are observed as kinks or discontinuities in the transmission and reflection spectra of the metamaterial and they are also known as Woods anomalies.20 The first order lattice mode (0,1) traps the diffracted field along the metallic interface of the periodic metamaterial array.21 Due to the trapped fields, the lattice mode behaves like a dark mode and its resonant frequency can be easily controlled by changing the periodicity of the array. Coupling of the lattice mode to the eigenmodes of the metamaterial can lead to intriguing effects such as extraordinary transmission22,23 and/or resonance line-width narrowing21,24,25 that give rise to extremely high Q-factors. We note that control over the Q-factor and depth of resonances have also been reported for three-dimensional metamaterials consisting of coupled resonators that can be tuned dynamically using MEMS approaches.26,27 The lattice modes, that exist in any periodic array, allow line-width and amplitude of resonances to be engineered while retaining the simplicity and low fabrication cost of planar metamaterials. High Q-factor devices minimize radiative losses and increase the sensitivity of the confined fields to external perturbation. These characteristics are needed in practical applications such as metamaterial spasers,28 sensors,8 and slow light devices.29

In this work, we investigate the coupling between the fundamental lattice mode and the hybridized resonances of a terahertz asymmetric split-ring resonator (TASR) metamaterial array with the incident electric field polarized parallel to the gap bearing arm of the resonator. We demonstrate Q-factor enhancement of an individual hybrid mode when it is resonantly coupled to the lattice mode and disappearance of the mode-splitting associated with decoupling of the lattice mode from the TASR eigenmodes. Hence, our results indicate that the mode splitting, also known as lattice induced transparency (LIT),30 requires coupling of the lattice mode

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to the TASR eigenmodes. These eigenmodes are the anti-symmetric resonance mode that is excited due to structural symmetry breaking and the symmetric resonance mode that is a broad dipole resonance occurring in both perfectly symmetric as well as asymmetric structures. Through this lattice mode coupling and a three-oscillator model, we show the existence of an anti-crossing behavior of the hybridized resonances that suggests a strong-coupling regime of the bright-lattice-dark (dipole-diffractive-asymmetric) mode interactions.

By varying the period $P$ of the metamaterial lattice, the frequency of the fundamental lattice mode can be tuned to couple to either of the two hybrid resonances. The frequencies of the lattice modes for a square lattice are given by

$$f_{LM} = \frac{c}{nP} \sqrt{I^2 + J^2},$$

where $c$ is the speed of light in vacuum, $n$ is the refractive index where the lattice mode propagates, $P$ is the lattice period, and $(i,j)$ are non-negative integers defining the order of the lattice mode. In our study, we consider coupling between the metamaterial resonances and the first-order lattice mode $(0,1)$ that propagates along the substrate of our metamaterial, for which $f_{LM} = c/nP$.

Figure 1(a) shows the metamaterial array that consists of periodically arranged asymmetric split-ring resonators. The dimensions of the metamaterial’s square unit cell are depicted in the inset of Fig. 1(b). The asymmetry, $d = 8 \mu m$, is introduced in the structure by moving one of the gaps away from the central vertical axis and the period $P$ of the lattice is varied to tune the frequency of the lattice mode. The metamaterial samples were fabricated using photolithography with positive photoresist (AZ5214E) on a double side polished, high resistivity ($> 5000 \ \Omega \cdot \text{cm}$) silicon substrate of 500 $\mu m$ thickness. A 200 $\mu m$ thick aluminium layer was thermally evaporated over the patterned substrate and the undesired aluminium was lifted off using acetone. The fabricated metamaterial arrays have an overall size of 1 cm $\times$ 1 cm each and an optical microscope image of a small portion of an array is shown in Fig. 1(b). The samples were characterized using photoconductive-antenna-based terahertz time domain spectroscopy (THz TDS) and the transmission amplitude was obtained by using the expression, $|t(\omega)| = |E_S(\omega)/E_R(\omega)|$, where $E_S(\omega)$ and $E_R(\omega)$ are Fourier transforms of the electric field transmitted through the metamaterial sample and a reference substrate, respectively. The metamaterial samples were loaded on a sample holder with a circular aperture of 10 mm diameter and characterized by a THz beam of 5 mm diameter, which corresponds to 25% of the exposed metamaterial area, see supplementary material Fig. S1. The illuminated area corresponds to 1400 to 4000 resonators for metamaterial periodicities varying from 120 $\mu m$ to 70 $\mu m$, respectively.

First, numerical investigations were performed using CST Microwave Studio Frequency Domain Solver to understand the effects of periodicity variation on the resonances of the TASR metamaterial. The aluminum split rings were modelled with a DC conductivity of $3.56 \times 10^7 \ \text{S/m}$ and the silicon substrate with a refractive index of $n = 3.42$. In the simulation, the periodicity $P$ of the structure was varied from 70 $\mu m$ to 120 $\mu m$ to investigate the interaction of the first order lattice mode with the hybridized modes of the TASR. Figure 2 shows both simulated and measured transmission spectra, which are in good qualitative agreement. Quantitative deviations in the line-widths and amplitude of the resonances between the

FIG. 2. (a)–(h) Experimental (black spheres) and simulated (red solid lines) transmission amplitude of the TASR metamaterial array with varying lattice period $P = 70, 75, 80, 82, 84, 87, 90, \text{ and } 120 \mu m$, respectively. The low frequency hybrid mode (LFHM), high frequency hybrid mode (HFHM) and lattice induced transparency (LIT) are indicated in (a). The fundamental lattice mode $(0,1)$ is marked by a dark blue triangle and the $(1,1)$ order lattice mode is indicated by a green triangle.
simulated and experimental results are mainly due to the low spectral resolution of the THz TDS setup. Deviations in the resonance frequencies of few percent are mainly due to fabrication imperfections. By increasing the period $P$ from 70 $\mu$m to 120 $\mu$m, the spectral position of the first order lattice mode is varied to couple to the metamaterial resonances. Through this coupling, the split resonances redshift, and their spectral line-width changes. Mode splitting (mode hybridization) with an intermediate transmission peak is observed. The resonant transmission amplitude minima and maxima of the metamaterial are the narrowest at critical periods, where the lattice mode resonantly couples with either of the hybrid modes [Figs. 2(b) and 2(f) or the transmission peak [Fig. 2(c)]. As the lattice periodicity is increased above 90 $\mu$m, the modesplitting disappears [Figs. 2(g) and 2(h)]. This indicates that coupling of the lattice mode to the metamaterial resonances in the TASR contributes to the hybridization of modes, which we address as the low frequency hybrid mode (LFHM) and the high frequency hybrid mode (HFHM). We note that the (1,1) order lattice mode can be seen for periodicities of 90 $\mu$m and 120 $\mu$m at frequencies of 1.38 THz and 1.03 THz, respectively [Figs. 2(g) and 2(h)].

As the lattice mode is swept across the metamaterial resonances, drastic changes occur in the linewidths and the resonance intensity of the LFHM, the HFHM and the LIT resonance spectra. Figures 3(c) and 3(d) show the FoM for the HFHM and LFHM resonances, which reached a maxima of 11 and 19, respectively. Like its $Q$-factor, the FoM of the HFHM is the largest at smaller periods. As the period increases from 65 $\mu$m to 87 $\mu$m, it drops by an order of magnitude due to the broadening of the resonance and the increase in resonant transmission intensity along with the vanishing of the neighboring LIT transmission peak at larger lattice periods. In the case of the LFHM, not only the $Q$-factor but also the FoM shows an exponential increase, which reaches a maximum FoM of 19 and $Q$-factor of 113 at $P = 87 \mu$m. At this period, the LFHM provides high sensitivity to perturbation and low radiative losses due to the strong coupling with the lattice mode that enhances the confined near-field energy in the split-ring gap.

Further investigation of the electric field confinement and surface currents of the lattice matched hybrid modes ($P = 75$ and 87 $\mu$m) is shown in Fig. 4. Little electric field confinement was seen in both hybrid modes at $P = 75 \mu$m as the surface currents remain in a dipole configuration which is highly radiative. However, at $P = 87 \mu$m, when the lattice mode frequency closely matches the LFHM, the surface currents are in a quadrupole configuration resulting in strong electric field confinement and a high $Q$-factor of 113 [Fig. 4(b), $P = 87 \mu$m, left]. Another point to note is that as the first order lattice mode crosses the LFHM, the surface currents and field distribution of the HFHM show a dipole configuration [Fig. 4(b), $P = 87 \mu$m, right], which corresponds to a broad minimum in transmission. Interestingly, for periods larger than 87 $\mu$m, the modesplitting is barely visible in the spectrum (Fig. 2), suggesting that the fundamental lattice mode decouples from the TASR eigenmodes and modesplitting switches off. This investigation indicates the role of the first order lattice mode in the modesplitting effect in the system. However, our structure also possesses a structural asymmetry that contributes to the observed modesplitting. Therefore, a similar investigation was performed with a symmetric resonator (symmetric gap, $d = 0$) of otherwise same dimensions. No mode splitting was observed in the

The resonant change in transmission becomes small at larger periodicities, see Fig. 2. This could be due to decoupling from the lattice mode and a weaker collective response from the larger unit cells in the metamaterial structure.31,32 On the other hand, as the lattice period is increased such that the lattice mode frequency approaches the LFHM, there is an exponential increase in its $Q$-factor, as shown in Fig. 3(b). This exponential increase is due to the gradual increase in coupling of the lattice mode to the HFHM that confines the electromagnetic energy in the metamaterial array. At $P = 87 \mu$m, where the lattice mode resonantly couples with the LFHM, the $Q$-factor reaches a maximum of 113, which is an order of magnitude higher than the HFHM of the same structure. In this case, both narrowing and reduced depth of the transmission resonance contribute to the observed exponential increase in the $Q$-factor. The practical importance of tailoring the $Q$-factors is apparent in terms of a figure of merit (FoM), which also considers the strength of the resonance and is useful for designing efficient sensors and nonlinear devices. The figure of merit ($FoM = Q \times \Delta f$) is defined as the product of $Q$-factor and change in transmission intensity $\Delta f$ (depth) of the resonance spectra. Figures 3(c) and 3(d) show the FoM for the HFHM and LFHM resonances, which reached a maxima of 11 and 19, respectively. As the lattice mode frequency falls below the HFHM, the HFHM resonance broadens and the associated transmission increases.

FIG. 3. $Q$-factor and figure of merit (FoM) of the HFHM and LFHM of the asymmetric metamaterial for different periodicities: (a) and (b) $Q$-factors of the HFHM and LFHM reaching a maximum of 31 and 113, respectively. (c) and (d) FoM of the HFHM and LFHM reaching a maximum of 11 and 19, respectively.
The coupled equations representing the interaction between the bright, lattice, and dark modes are expressed as

\[
\begin{align*}
\left(-\omega^2 - i\omega\gamma_b + \omega_b^2\right)\ddot{x}_b + \Omega_b^2\dot{x}_{LM} &= \ddot{E}(\omega), \\
\left(-\omega^2 - i\omega\gamma_{LM} + \omega_{LM}^2\right)\ddot{x}_{LM} + \Omega_{LM}^2\dot{x}_b + \Omega_b^2\dot{x}_d &= 0, \\
\left(-\omega^2 - i\omega\gamma_d + \omega_d^2\right)\ddot{x}_d + \Omega_d^2\dot{x}_{LM} &= 0,
\end{align*}
\]

where \((\ddot{x}_b, \ddot{x}_{LM}, \ddot{x}_d), (\omega_b, \omega_{LM}, \omega_d),\) and \((\gamma_b, \gamma_{LM}, \gamma_d)\) are the displacement amplitudes, resonance angular frequencies, and damping rates of the bright mode, lattice mode, and dark mode, respectively. \(\Omega_b\) and \(\Omega_d\) are the bright-lattice and dark-lattice mode coupling strengths, respectively. In this model, the lattice mode mediates the coupling between the bright and dark modes. Equation (2a) is the equation of motion for the bright mode and it is driven by the incident light, while (2c) is the equation of motion of the dark mode of the TARS, and they are both coupled to the lattice equation of motion (2b) by the coupling strengths \(\Omega_b\) and \(\Omega_d\), respectively.

The bright-mode frequency \(\omega_b\) is taken as the resonance frequency of a symmetric split-ring resonator of 1.1 THz, whereas \(\omega_{LM}\) is obtained from the simplified expression of Eq. (1). Also \(\omega_d\) is taken to be equal to \(\omega_{LM}\) as the lattice mode mediates the coupling of the dark mode to the bright mode in our model. Based on the line-widths, the damping rates were taken as \(\gamma_b = 10^{12}\) rad/s and \(\gamma_{LM} = \gamma_d = 0.15 \times 10^{12}\) rad/s, while \(\Omega_b\) and \(\Omega_d\) are taken as free parameters. With this model, the eigenvalues of the coupled equations are solved through the diagonalization of matrix \(M\)

\[
M = \begin{pmatrix}
\Lambda_b & \Omega_b^2 & 0 \\
\Omega_b^2 & \Lambda_{LM} & -\Omega_d^2 \\
0 & \Omega_d^2 & \Lambda_d
\end{pmatrix},
\]

where \(\Lambda_i = -\omega^2 - i\omega\gamma_i + \omega_i^2\) and \(i = b, LM, d\). The eigenvalues of the diagonalized matrix are the two hybrid modes of the coupled system and the lattice mode. A numerical fitting was performed to obtain the best values to fit the simulations, resulting in \(\Omega_b = 2.30 \times 10^{12}\) rad/s and \(\Omega_d = 1.84 \times 10^{12}\) rad/s. As illustrated in Fig. 5, the three-oscillator model (red circles) fits the simulations (green squares) well at larger periodicities. But the oscillator model does not account for the nearest neighbor coupling of the metamaterial unit cells, resulting in deviations at smaller periodicities, where the simulated resonance shift of the HFHM is less prominent than predicted by the analytical model. Figure 5 also shows the crossing of the period-dependent lattice mode and the bright dipole mode at 1.1 THz. This crossing corresponds to the high-\(Q\) lattice induced transparency peak shown in Fig. 2(c). In this system, the lattice mode mediates the coupling of the bright mode to the dark mode, giving rise to the observed anti-crossing. Therefore, this anti-crossing behavior alongside the coupling strength where \(\Omega > \gamma\) is a signature of a strongly coupled system that can only be observed at periodicities \(P < 90\) µm as the mode-splitting vanishes at larger periodicities.

In summary, we demonstrated how lattice modes can be used to control the intensity and line-width of hybridized
resonances in metamaterial arrays of asymmetric resonators. By varying the metamaterial periodicity, we selectively coupled the fundamental lattice mode to either of the hybridized resonances of terahertz asymmetric split-ring resonators and were able to tune the resonance $Q$-factor of an individual hybrid mode from 5 to 113. We observed strong coupling between the lattice mode and the hybridized resonances in the form of an anti-crossing behavior using a three-oscillator model. The strong coupling also leads to an intensified mode-splitting behavior giving rise to lattice induced transparency. This simple approach of in-plane coupling of the fundamental lattice mode and metamaterial resonances can be used as an efficient technique to optimize metamaterial designs to reduce radiative losses and to provide high-$Q$ resonances for practical applications such as ultrasensitive sensors and low threshold lasing spasers.

See supplementary material for the transmission spectra of the metamaterials consisting of symmetric resonators and detailed information on the coupled oscillator model.

The authors acknowledge research funding support from the Singapore National Research Foundation (NRF), the French National Research Agency (ANR, Grant No. NRF2016-NRF-ANR004), and the UK’s Engineering and Physical Sciences Research Council (EPSRC, Grant No. EP/M009122/1). The data from this paper is available at the University of Southampton ePrints research repository: http://doi.org/10.5258/SOTON/D0397.